1. [1pt each]

(a) Let $f(z) = \frac{z}{\bar{z}}$. Is $f$ continuous everywhere?

(b) Let $z = x + iy$ and let $f(z) = 3xy + i(x - y^2)$. Find $\lim_{z \to 3+2i} f(z)$.

(c) Let $z_n = 1 + i(1 - \frac{2}{n})$. Find $\lim_{n \to \infty} z_n$. Find $\lim_{n \to \infty} z_n^3$.

(d) Let $z_n = \text{Arg}(-1 + \frac{i}{n})$. Find $\lim_{n \to \infty} z_n$.

(e) $\lim_{n \to \infty} (-i)^n$.

2. [2pts] Suppose that $f : \mathbb{C} \to \mathbb{C}$ is a function which satisfies $f(z) = f(\bar{z})$ for all $z \in \mathbb{C}$ and which is continuous at $z = 0$. Show that $f$ is a constant function.

3. [1pt each] For each of the following functions $f$, describe the domain where they are defined and compute their derivatives. (You can use the derivative rules – there is no need to differentiate the functions using the limit definition.)

(a) $f(z) = z^4 - z e^z - 4\pi$.

(b) $f(z) = \left(z + \frac{1}{z}\right)^{100}$.

(c) $f(z) = \frac{z^2}{e^{2z} - 2}$.

(d) $f(z) = \frac{az + b}{cz + d}$ (with $ad - bc \neq 0$).

4. [1pt each] For each of the following functions $u(x, y)$ and $v(x, y)$, check if the Cauchy-Riemann equations hold. If they do hold, find a function $f(z)$ so that $f(x + iy) = u(x, y) + iv(x, y)$.

(a) $u(x, y) = y$, $v(x, y) = -x$.

(b) $u(x, y) = 5x - 3y$, $v(x, y) = 3x + 5y$.

(c) $u(x, y) = \cos(x) \sin(y)$, $v(x, y) = \sin(x) \cos(y)$.

(d) $u(x, y) = x^3 - 3xy^2 + 1$, $v(x, y) = 3x^2y - y^3$.

5. [2pts] Let $f : \mathbb{C} \to \mathbb{C}$ be an entire function. Define a new function $g(z)$ by

$$g(z) = \overline{f(z)}.$$
(i.e., given a complex number $z$, to compute $g(z)$ we input the complex number $\bar{z}$ into $f$, and then take the complex conjugate of the answer.)

Is $g$ also an entire function? (Prove this or give a counterexample).

6. [1pt each]

(a) Let $f(z) = |z|^2$. Find all points where $f$ is differentiable.

(b) Let $f(z) = \log(z)$. Find a domain $D$ where $f$ is differentiable, and find its derivative.

(c) Let $f(z) = z^\alpha$ where $\alpha \in \mathbb{R}$. Find a domain $D$ where $f$ is differentiable, and find its derivative.