1. [2 pts] Let \( z_0 \in \mathbb{C} \) be a fixed complex number, and for any real number \( r > 0 \) let \( \gamma_r \) be the circle of radius \( r \) around \( z_0 \), oriented counterclockwise. Find \( \int_{\gamma_r} \frac{1}{z - z_0} \, dz \).

2. [2pts each]

For a point \( z = x + iy \in \mathbb{C} \) let \( \gamma_1 \) be the path that goes in a straight line from 0 to \( z \), and let \( \gamma_2 \) be the path that goes from 0 to \( x \), and then to \( z \), as illustrated below.

Let \( f(z) = 3|z|^2 \).

(a) Find \( \int_{\gamma_1} f(z) \, dz \).

(b) Find \( \int_{\gamma_2} f(z) \, dz \).

The answers in (a) and (b) depend on \( z = x + iy \) and so define complex functions \( F_1(z) \) and \( F_2(z) \).

(c) Is the function \( F_1 \) from (a) analytic (holomorphic)?

(d) Is the function \( F_2 \) from (b) analytic (holomorphic)?

3. [2pts each] Show the following estimates for questions (a),(b),(c):

(a) \( \left| \int_{\gamma} \frac{dz}{z^2 - i} \right| \leq \frac{3\pi}{4} \), where \( \gamma = \{ z \in \mathbb{C} \mid |z| = 3 \} \), oriented counterclockwise.

(b) \( \left| \int_{\gamma} \log(z) \, dz \right| \leq \frac{\pi^2}{4} \), where \( \gamma = \{ z \in \mathbb{C} \mid |z| = 1, \ 0 \leq \text{Arg}(z) \leq \frac{\pi}{2} \} \), oriented counterclockwise.

(c) \( \left| \int_{\gamma} \exp(\sin z) \, dz \right| \leq 1 \), where \( \gamma \) is the straight line segment from \( z = 0 \) to \( z = i \).
(d) Estimate an upper bound of \( \left| \int_{|z|=3} \frac{\Log(z)}{z - 4i} \, dz \right| \).

4. [1pt each]
   (a) Let \( \gamma \) is the quarter-circle centered at the origin and extending from 2 to \( 2i \). Parametrize \( \gamma \) and compute
   \[
   \int_{\gamma} (z^2 - 3|z| + \Im(z)) \, dz.
   \]

   (b) Let \( \gamma \) be a directed smooth curve with initial point \( \alpha \) and the endpoint \( \beta \). Show that
   \[
   \int_{\gamma} z \, dz = \frac{\beta^2 - \alpha^2}{2}.
   \]

   (c) Suppose that \( \gamma \) is any closed contour. Show that
   \[
   \int_{\gamma} z \, dz = 0.
   \]

   (d) Let \( \gamma \) is a smooth closed contour. Compute
   \[
   \int_{\gamma} z^n \, dz \quad \text{for any} \quad n = 0, 1, \ldots
   \]

   (e) Let \( D = \mathbb{C} \setminus \{(x,0)|x \leq 0\} \). Let \( \gamma \) be a contour lying in \( D \) with the initial point \( 1 \) and the terminal point \( \alpha \) where \( \alpha \in D \) is a fixed complex number. Compute
   \[
   \int_{\gamma} \frac{dz}{z}.
   \]