Problem 1:

a) Compute the (two-sided) Z-transform of
\[ x(n) = 4^n 1_{(n \geq 0)} \]

b) Compute the (two-sided) Laplace-transform of
\[ x(t) = e^{3t} 1_{(t \geq 0)} \]

Find the regions in the complex plane, where the transforms are finite valued.

c) Show that the one-sided Laplace transform of \( \cos(\alpha t) \) satisfies
\[ \mathcal{L}_+\{\cos \alpha t\} = \frac{s}{s^2 + \alpha^2}, \quad \text{Re}(s) > 0 \]

d) Compute the inverse Laplace transform of
\[ \frac{s^2 + 9s + 2}{(s - 1)^2(s + 3)}, \quad \text{Re}(s) > 1 \]

Hint: Use partial fraction expansion and the properties of the derivative of a Laplace transform.

Problem 2:

Suppose that \( x \) is a sequence which has non-zero values only on the positive integers. Explain whether the following is correct or not: If the circle \( \{ z : |z| = r \} \) is in the Region of Convergence for the Z-transform of \( x \), then, \( \{ z : |z| > r \} \) is also in the region of convergence.

Problem 3:

Find the inverse Z-transform of:
\[ X(z) = \frac{3 - \frac{5}{6} z^{-1}}{(1 - \frac{1}{4} z^{-1})(1 - \frac{1}{3} z^{-1})}, \quad |z| > 1 \]

Problem 4:

Let \( x(n) = 0 \) for \( n < 0 \) and \( x(n) \leq Mr^n \) for some \( M, r \). Show that
\[ \lim_{|z| \to \infty} X(z) = x(0) \]
Problem 5:

Let $P(z)$ and $Q(z)$ be polynomials in $z$. Let the transfer function of a discrete-time LTI system be given by

$$H(z) = \frac{P(z)}{Q(z)}$$

a) Suppose the system is BIBO stable. Show that the system is causal (non-anticipative) if and only if $P(z)$ is a proper fraction (that is the degree of the polynomial in the numerator cannot be greater than the one of the denominator).

b) Show that the system is BIBO stable if and only if the Region of Convergence of the transfer function contains the unit circle. Thus, for a system to be both causal and stable, what are the conditions on the roots of $Q(z)$?