MATH 337

Markov Chains

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Mods to consider for the notquitedrunkardswalk

- Iterate over all initial states
- Iterate over (at least 1000) samples
- Stop time iterations when state = home or state = jail
- Keep track of ‘successfully making it home/jail’
Computing $\mathbb{E}(X_1)$ ...

$$f_{X_1,X_2}(s,t) = \frac{1}{2\pi \sigma_1 \sigma_2 \sqrt{1-p^2}} \exp \left( -\frac{1}{2\pi (1-p)^2} \left( \left( \frac{s-\mu_1}{\sigma_1} \right)^2 - 2p \left( \frac{s-\mu_1}{\sigma_1} \right) \left( \frac{t-\mu_2}{\sigma_2} \right) + \left( \frac{t-\mu_2}{\sigma_2} \right)^2 \right) \right)$$

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} s f_{X_1,X_2}(s,t) dt \, ds$$

U-substitution (translation) so that $-Ct^2 + Dt \Rightarrow -E(t - F)(t + F)$

$$N(A, b) \Rightarrow \frac{1}{\sqrt{2\pi b}} \int_{-\infty}^{\infty} \exp \left( -\frac{(x + A)^2}{b^2} \right) = 1$$
Classification of Markov states

- Accessible
- Communicative
  \[ i \leftrightarrow j \iff j \leftrightarrow j \]
  \[ i \leftrightarrow j \text{ and } j \leftrightarrow k \iff i \leftrightarrow k \]
- Irreducible
  States that communicate form a class
- Recurrent
- Transient
- Absorbing
Will a stochastic process ever return to its current state?

Denote $f_{ii}$ as the prob of returning to state $i$ starting from state $i$.

1) $f_{ii} = 1$ (Recurrent)

2) $f_{ii} < 1$ (Transient)
Ex. Drunkard’s walk (M=5)

- $f_{00} =$
- $f_{44} =$
- $f_{22} =$
Expected # of time periods in (transient) state i

Consider:

1 current period

\( f_{ii} \) next period

\( f_{ii} \cdot f_{ii} \) next-next period

etc
Periodicity & First passage times

Period of state $i$ is the largest (integer) $d$ s.t.

$$p_{ii}^{(n)} = 0 \text{ whenever } n \text{ is not divisible by } d.$$