MATH 337

Markov Chains

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Summary

- $p_{ij}$ — transition prob of going from state i to state j in one time step
- $f_{ii}$ — prob of returning to state i given currently in state i
- $f_{ij}^{(n)}$ — prob of going from state i to state j in exactly n steps
- $\mu_{ij}$ — Expected ‘first passage’ time from i to j
- $\pi_j$ — Steady state prob of state j
Ex.

Suppose we have the transition matrix given by:

\[
P = \begin{pmatrix} a_1 & b_1 \\ a_2 & b_2 \end{pmatrix}
\]

What is the prob of returning to state 0 given that you start at state 0?
Ex.

Suppose we have the transition matrix given by:

\[ P = \begin{pmatrix} a_1 & b_1 \\ a_2 & b_2 \end{pmatrix} \]

What is the prob of moving to state 0 in 3 steps given that you start at state 0?
Ex.

Suppose we have the transition matrix given by:

\[ P = \begin{pmatrix}
  a_1 & b_1 \\
  a_2 & b_2
\end{pmatrix} \]

What is the expected first passage time from state 0 to state 1?
Ex.

Suppose we have the transition matrix given by:

\[
P = \begin{pmatrix} a_1 & b_1 \\ a_2 & b_2 \end{pmatrix}
\]

What is the long-term probability?
Continuous time Markov Chains

Defn summary:

\[ P(X(t + s) = i | X(s) = i) = P(X(t) = i | X(0) = i) \]

\[ \Updownarrow \]

\[ P(T_i > t + s | T_i > s) = P(T_i > t) \]

(with markov prop, and t is indep of s)