MATH 337

Linear Programs

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Linear Program (Standard form)

\[
\max \sum_{i=1}^{n} c_i x_i \\
\text{s.t.} \\
A \cdot \vec{x} \leq \vec{b} \\
x_i \geq 0
\]

\(A\) is an \(m \times n\) real valued matrix
\(\vec{b}\) is an \(m \times 1\) vector
\(\vec{x}\) is \((x_1, x_2, \ldots, x_n)^T\)
Ex.

Find $a$ and $c$ such that the largest of $|2 - a - c|$, $|4 - 3a - c|$, and $|7 - 4a - c|$ is as small as possible.
Linear Program (Canonical form)

\[
\begin{align*}
\text{max} & \sum_{i=1}^{n} c_i x_i \\
\text{s.t.} & \quad A \cdot \vec{x} = \vec{b} \\
& \quad x_i \geq 0
\end{align*}
\]

\(A\) is an \(m \times n\) real valued matrix
\(\vec{b}\) is an \(m \times 1\) vector
\(\vec{x}\) is \((x_1, x_2, \ldots, x_n)^T\)
Converting between standard and canonical forms

Slack variables:

e.g.

\[x_1 + x_2 \leq 70 \Rightarrow x_1 + x_2 + s_1 = 70\]

where \(s_1 \geq 0\)

e.g.

\[x_1 + x_2 \geq 70 \Rightarrow x_1 + x_2 - s_1 = 70\]

where \(s_1 \geq 0\)
LP in general

An LP in general has either:

1. Unique optimal solution
2. Multiple optimal solutions
3. Is infeasible (no feasible solution)
4. Unbounded
Solution techniques: Graphical

Two approaches:

i) Corner points (Extreme points)

ii) Iso-value line(s)
Solution techniques: Graphical

Ex.

\[
\begin{align*}
\text{max } & \quad 3x_1 + 2x_2 \\
\text{s.t. } & \quad x_1 + x_2 \leq 80 \\
& \quad 2x_1 + x_2 \leq 100 \\
& \quad x_1 \leq 40 \\
& \quad x_i \geq 0
\end{align*}
\]
Solution techniques:

Fourier-Motzkin Elimination

Setup: convert to standard form, make new inequality $z \geq \text{obj fun}$

1) Normalize $x_1$ in each inequality
2) Eliminate $x_1$
3) Repeat for $x_2, ...$
4) Determine smallest $z$ that satisfies inequalities
5) Backwards substitute to find $x_i's$