MATH 337
Linear Programs

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Course website: http://www.mast.queensu.ca/~math337/index.shtml
Project ideas

- Stochastic processes
  - Waiting in line at Tim’s for coffee
  - Elaborate on crosswalk problem
  - Player assist per Mathew’s goal

- Mathematical Programs
  - Supply/demand
    - Medical supply distribution during Ebola outbreak
  - Traffic flow
  - Non-cooperative games

- Theoretical
  - Gradient flows
  - Complementarity systems / Projected DEs
  - Variational Inequalities
Solution techniques: Graphical

Two approaches:

i) Corner points (Extreme points)

ii) Iso-value line(s)
LP in general

An LP in general has either:

1. Unique optimal solution
2. Multiple optimal solutions
3. Is infeasible (no feasible solution)
4. Unbounded
Solution techniques:

Fourier-Motzkin Elimination

Setup: convert to standard form, make new inequality $z \geq obj \ fun$

1) Normalize $x_1$ in each inequality
2) Eliminate $x_1$
3) Repeat for $x_2$, ...
4) Determine smallest $z$ that satisfies inequalities
5) Backwards substitute to find $x_i's$
Ex.

\[ \text{min } 2x_1 + 2x_2 + 3x_3 \]
\[ \text{s.t. } \]
\[ x_1 + x_2 + x_3 \leq 2 \]
\[ 2x_1 + x_2 \leq 3 \]
\[ 2x_1 + x_2 + 3x_3 \geq 3 \]
\[ x_i \geq 0 \]
The Simplex Algorithm

Setup:
- Start with initial feasible solution
- Right obj function as constraint $z - \text{obj fun} = 0$
- Write LP in canonical form (as a max prob)

Rule 1:
- If all $x_i's$ in z-constraint are non-negative STOP
  - You are at an optimal soln
- Select an $x_i$ in z-constraint with negative coefficient
The Simplex Algorithm

- Select an $x_i$ in z-constraint with negative coefficient

- ‘Pivot’ with this variable to make $x_i$ disappear in other equations
The Simplex Algorithm

Rule 2

- Always pivot in the row that has the smallest ratio of 'entering' variable and RHS value

Repeat until coefficients in z-constraint are non-negative
<table>
<thead>
<tr>
<th></th>
<th>$x_{12}$</th>
<th>$x_{13}$</th>
<th>$x_{23}$</th>
<th>$x_{24}$</th>
<th>$x_{25}$</th>
<th>$x_{34}$</th>
<th>$x_{35}$</th>
<th>$x_{45}$</th>
<th>$x_{53}$</th>
<th>RHS</th>
</tr>
</thead>
<tbody>
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<td>20</td>
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<tr>
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<td>1</td>
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<td>1</td>
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<tr>
<td>Node 3</td>
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<tr>
<td>Node 4</td>
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<td>-1</td>
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<td>-1</td>
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<td>1</td>
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<td>-5</td>
</tr>
<tr>
<td>Node 5</td>
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<td>1</td>
<td></td>
<td>-15</td>
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<tr>
<td>Cap</td>
<td>15</td>
<td>8</td>
<td>$\infty$</td>
<td>4</td>
<td>10</td>
<td>15</td>
<td>5</td>
<td>$\infty$</td>
<td>4</td>
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</tr>
<tr>
<td>Obj fun ($$)</td>
<td>4</td>
<td>4</td>
<td>2</td>
<td>2</td>
<td>6</td>
<td>1</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>(min)</td>
</tr>
</tbody>
</table>