MATH 337

Games, Networks, and IP

Course instructor: Dr. Scott Greenhalgh
Email: scott.greenhalgh@queensu.ca
Office: Jeff 516

Course website: http://www.mast.queensu.ca/~math337/index.shtml
Vaccination games

- Continuous games
- Nash equilibria
- Variational inequalities
- Projected differential equations
- Vaccination games
  - Through Proj DEs, we can study how vacc adjusts
Variational Inequality:

Defn. Let $K \in \mathbb{R}^n$ be a closed convex and bounded set, and $F: K \to \mathbb{R}^n$ a mapping. Then a VI is the problem:

$$\text{find } x \text{ s.t. } \langle F(x), y - x \rangle \geq 0 \forall y \in K$$
Games, VI, Projected DEs

If $x^*$ is a Nash equilibrium then

$$\langle F(x^*), x - x^* \rangle \geq 0 \quad \forall x \in K$$

For $F(x^*) = \langle -\nabla U_1(x), \ldots - \nabla U_m(x) \rangle$

The solution to a VI(K,F) corresponds to the critical points of the DE:

$$\frac{dx}{dt} = \Pi_K(x, -F(x))$$

*given $U_i$ is $C^1$ & concave w.r.t. $x_i$
Setup: Vaccination and the theory of games

- Individuals strategy is the probability $P$ that they vaccinate

- Morbidity from vaccination $r_v$

- Morbidity from infection $r_i$

- Probability of unvaccinated becoming infected, given vaccination level $p$, is $\pi_p$
Setup: Vaccination and the theory of games

Utility function:

\[ U(P_i, p) = -r_v P_i - r_{inf} \pi_p (1 - P_i) \]
The Vaccination game

Details:

- You are immortal

- You have 5 children every generation

- Every generation is at risk of a deadly disease

- You have the options too:

  A) Vaccinate all you children (at the cost of loosing 1 child due to vaccine complications)

  B) Do nothing

- The immortal that accumulates the most children is the winner
Details:

- Background infection rate
  - Susceptible can start off infected

- Everyone ‘interacts’ with everyone
  - Vaccinated cannot spread disease/become infected
  - Susceptible interacting with infected has a particular probability to become infected

- At the end of a generation, those who are not infected get 5 children (minus 1 if they vaccinate)

- The vaccination level (from a random sample ~25%) is shared