In class today you asked us to come up with some rationale of how the authors of the one-card, two round poker with bluffing determined that they could assume that given player 1 receives the high card their payoff from any of bb, bf or f were all equal to the payoff from bb. I have an explanation for how the authors arrived at this outcome but it requires us to take a step back from the game and to re-evaluate how player 1 determines their strategy.

In class you presented three possible strategies for player 1, they could bb, bf or f. Instead, let’s assume player 1 determines their strategy both for what they would do if they receive the high card or if they receive the low card before the card is dealt. If we think about the strategies this way player 1 actually has 9 different possible strategies:

1) if high card: bb, if low card: bb
2) if high card: bb, if low card: bf
3) if high card: bb, if low card: f
4) if high card: bf, if low card: bb
5) if high card: bf, if low card: bf
6) if high card: bf, if low card: f
7) if high card: f, if low card: bb
8) if high card: f, if low card: bf
9) if high card: f, if low card: f

Now if we assume that player 1 receives the high card with probability $\frac{1}{2}$ we can create an expected payoff matrix:

<table>
<thead>
<tr>
<th></th>
<th>bb</th>
<th>bf</th>
<th>f</th>
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<tbody>
<tr>
<td></td>
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</table>
\[
\begin{array}{|c|c|c|c|}
\hline
& bb,bb & 0.5 \times 6 + 0.5 \times (-6) = 0 & 4 & 2 \\
\hline
bb,bf & 1 & 0 & 2 \\
\hline
bb,f & 2 & 1 & 0 \\
\hline
bf,bb & -5 & 0 & 2 \\
\hline
bf,bf & -4 & -4 & 2 \\
\hline
bf,f & -3 & -3 & 0 \\
\hline
f,bb & -4 & 1 & 0 \\
\hline
f,bf & -3 & -3 & 0 \\
\hline
f,f & -2 & -2 & -2 \\
\hline
\end{array}
\]

(Note: I only listed the payoff to player 1 because the payoff to player 2 is always that number times negative one) Looking at this matrix we can see that player 1s strategy of bb,bb will (at least weakly) dominate all of the strategies from bf,bb and below. So this implies player 1 will only play a strategy that starts with bb. This makes sense given that if you are holding the high card you will only bet and never fold because you know you are going to win.

By iterated deletion of dominated strategies we can rewrite this payoff matrix as:

\[
\begin{array}{|c|c|c|c|}
\hline
& bb & bf & f \\
\hline
bb,bb & 0 & 4 & 2 \\
\hline
bb,bf & 1 & 0 & 2 \\
\hline
bb,f & 2 & 1 & 0 \\
\hline
\end{array}
\]
This is the expected payoff matrix we used in class to determine our probabilities in the MSNE. The authors may have used shorthand and removed the bb from the start of player 1's strategies just for simplicities sake and just left the part of what player 1 would do given they receive the low card.

This change makes a lot more sense when considering the probabilities we found of how often to play each move. It now breaks down as follows: if you receive the high card play bb every time, if you receive the low card play bb with probability 1/5 and play bf and f each with probability 2/5. This makes more sense that you would end up folding more often given you are holding the low card and you know you will lose big if you arrive at the part of the game where the card is revealed.