Consider the following matrix:

\[
\begin{pmatrix}
D & E & F \\
A & (9,-4) & (4,-2) & (-2,1) \\
B & (-2,4) & (5,3) & (0,2) \\
C & (5,2) & (2,4) & (1,5)
\end{pmatrix}
\]

Notice that this matrix has no dominant strategies for either player. We can use the theory of mixed strategy Nash equilibria to find a MSNE where each player uses all three of their moves at least some of the time. We begin by finding the expected payoffs for player 1 for moves A, B, and C. Let \(p_D, p_E, p_F\) be the probability player 2 uses move D, E, F, respectively.

\[
\begin{align*}
\pi_1(A, \sigma_2) &= 9p_D + 4p_E - 2p_F \\
\pi_1(B, \sigma_2) &= -2p_D + 5p_E + 0p_F \\
\pi_1(C, \sigma_2) &= 5p_D + 2p_E + 1p_F
\end{align*}
\]

Set \(\pi_1(A, \sigma_2) = \pi_1(C, \sigma_2)\) and \(\pi_1(B, \sigma_2) = \pi_1(C, \sigma_2)\), giving us the two equations:

\[
\begin{align*}
4p_D + 2p_E - 3p_F &= 0 \\
-7p_D + 3p_E - p_F &= 0
\end{align*}
\]

plus the last equation where \(p_D + p_E + p_F = 1\), since the probabilities associated with all of the moves must sum to 1. We now have a 3 by 3 linear system, which can be solved in a variety of ways. Use whichever technique you are most comfortable with, but here is how I would do it, using row reduction:

\[
\begin{pmatrix}
4 & 2 & -3 & | & 0 \\
-7 & 3 & -1 & | & 0 \\
1 & 1 & 1 & | & 1
\end{pmatrix}
\rightarrow
\begin{pmatrix}
4 & 2 & -3 & | & 0 \\
0 & 26 & -25 & | & 0 \\
0 & -2 & -7 & | & -4
\end{pmatrix}
\rightarrow
\begin{pmatrix}
4 & -2 & 3 & | & 0 \\
0 & 26 & -25 & | & 0 \\
0 & 0 & -116 & | & -52
\end{pmatrix}
\]

If you have forgotten how to row reduce, [http://www.math.tamu.edu/~fnarc/psfiles/rowred2012.pdf](http://www.math.tamu.edu/~fnarc/psfiles/rowred2012.pdf) is not a bad website. From here we can solve for the probabilities: \(p_D = \frac{7}{58}, p_E = \frac{29}{58}, p_F = \frac{13}{29}\).

Now we can use a similar process involving player 2’s expected payoffs. Let \(p_A, p_B, p_C\) be the probability player 2 uses move A, B, C, respectively.

\[
\begin{align*}
\pi_2(\sigma_1, D) &= -4p_A + 4p_B + 2p_C \\
\pi_2(\sigma_1, E) &= -2p_A + 3p_B + 4p_C \\
\pi_2(\sigma_1, F) &= p_A + 2p_B + 5p_C
\end{align*}
\]
Set $\pi_2(\sigma_1, D) = \pi_2(\sigma_1, F)$ and $\pi_2(\sigma_1, E) = \pi_2(\sigma_1, F)$, giving us the two equations:

$$5p_A - 2p_B + 3p_C = 0$$
$$3p_A - p_B + p_C = 0$$

plus the last equation where $p_A + p_B + p_C = 1$, since the probabilities associated with all of the moves must sum to 1. Same deal as last time for the solution:

$\begin{pmatrix} 5 & -2 & 3 & 0 \\ 3 & -1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 5 & -2 & 3 & 0 \\ 0 & -1 & 4 & 0 \\ 0 & -7 & -2 & -5 \end{pmatrix} \rightarrow \begin{pmatrix} 5 & -2 & 3 & 0 \\ 0 & -1 & 4 & 0 \\ 0 & 0 & 30 & 5 \end{pmatrix}$

From here we can solve for the probabilities: $p_A = \frac{1}{6}, p_B = \frac{2}{3}, p_C = \frac{1}{6}$.

Therefore our MSNE is $(\sigma_1, \sigma_2) = \left( \frac{1}{6} A + \frac{2}{3} B + \frac{1}{6} C, \frac{7}{58} D + \frac{25}{58} E + \frac{13}{29} F \right)$.

From this point we can calculate the expected payoffs for both players using the mixed strategy profile at the NE.

**Calculating Expected Payoff**

Basically, we sum the probability of every possible combination multiplied by its payoff. For player 1, this looks like:

$$\pi_1(\sigma_1, \sigma_2) = p_{AD} \pi_1(A, D) + p_{AE} \pi_1(A, E) + p_{AF} \pi_1(A, F) + \cdots + p_{CF} \pi_1(C, F)$$

$$= \left( \frac{1}{6} \right) \left( \frac{7}{58} \right) 9 + \left( \frac{1}{6} \right) \left( \frac{25}{58} \right) 4 + \left( \frac{1}{6} \right) \left( \frac{13}{29} \right) (-2) + \cdots + \left( \frac{1}{6} \right) \left( \frac{13}{29} \right) 1 = 1.913793103$$

For player 2, this looks like:

$$\pi_2(\sigma_1, \sigma_2) = p_{DA} \pi_2(A, D) + p_{DB} \pi_2(B, D) + p_{DC} \pi_2(C, D) + \cdots + p_{CF} \pi_2(C, F)$$

$$= \left( \frac{1}{6} \right) \left( \frac{7}{58} \right) (-4) + \left( \frac{1}{6} \right) \left( \frac{25}{58} \right) 4 + \left( \frac{1}{6} \right) \left( \frac{13}{29} \right) 2 + \cdots + \left( \frac{1}{6} \right) \left( \frac{13}{29} \right) 5 = 2.333333333$$

Since $\pi_1(\sigma_1, \sigma_2) < \pi_2(\sigma_1, \sigma_2)$, we would say this game favours Player 2.