Assignment 4  
Due 03/02/2012  

Pick one question from the first set and one from the second set.  

1. Find the MSNE for the following modified rock-paper-scissor games. Recall that the MSNE for the (unmodified) rock-paper-scissors game is: play each strategy 1/3 of the time. Before you find the MSNE for the following questions, look at how the payoffs deviate from the standard RPS setup and try to reason what the MSNE should be.  

(a) Paper is a wimpy move and the payoff one receives should reflect this:  

\[
\begin{array}{ccc}
R & P & S \\
R & 0,0 & -1/2,1/2 & 1,-1 \\
P & 1/2,-1/2 & 0,0 & -1,1 \\
S & -1,1 & 1,-1 & 0,0 \\
\end{array}
\]

(b) Rock is clearly superior:  

\[
\begin{array}{ccc}
R & P & S \\
R & 0,0 & -1,1 & 2,-1 \\
P & 1,-1 & 0,0 & -1,1 \\
S & -1,2 & 1,-1 & 0,0 \\
\end{array}
\]

(c) Also, rock can sharpen scissors, if used carefully:  

\[
\begin{array}{ccc}
R & P & S \\
R & 0,0 & -1,1 & 2,0 \\
P & 1,-1 & 0,0 & -1,1 \\
S & 0,2 & 1,-1 & 0,0 \\
\end{array}
\]

(d) One thing that’s always annoyed me about RPS is “paper covers rock”. Who cares? Is that actually losing? To be fair, paper could cover paper.  

\[
\begin{array}{ccc}
R & P & S \\
R & 0,0 & -1,1 & 1,-1 \\
P & 1,-1 & 1/2,1/2 & -1,1 \\
S & -1,1 & 1,-1 & 0,0 \\
\end{array}
\]

(e) Even though scissors may cut paper, paper will survive another day, but be half its strength:  

\[
\begin{array}{ccc}
R & P & S \\
R & 0,0 & -1,1 & 1,-1 \\
P & 1,-1 & 0,0 & -1/2,1 \\
S & -1,1 & 1,-1/2 & 0,0 \\
\end{array}
\]
2. Two firms produce comparable products. The payoff (profit) to firm one is \( \pi_1(q_1, q_2) = q_1 \cdot P_1(q_1, q_2) - C_1(q_1) \) and the payoff to firm two is \( \pi_2(q_1, q_2) = q_2 \cdot P_2(q_1, q_2) - C_2(q_2) \), where \( q_1 \) and \( q_2 \) are the quantities produced by firm one and firm two, respectively. The following questions pertain to this general setup.

(a) Suppose firm one is a subsidiary of Apple and is, therefore, able to overcharge for their product. This yields \( P_1(q_1, q_2) = (500 - q_1 - q_2) \), while \( P_2(q_1, q_2) = (400 - q_1 - q_2) \). Also, assume both firms have fixed costs \( c = 14 \) of production. That is, \( C_1(q_1) = 14q_1 \) and \( C_2(q_2) = 14q_2 \). If both firms simultaneously declare a quantity they will produce, find the optimal quantities. What prices can they charge?

(b) Using the same setup as in (a), suppose both firms decide to produce the same amount of the product. In order to accomplish this, firm one bends the truth a bit and informs firm two that their (firm one’s) price function will be the same as firm two’s, \( P_2(q_1, q_2) = (400 - q_1 - q_2) \). What quantity should they produce? What price will firm two be able to charge? Is it worth it for firm one to revert to their original price function, \( P(q_1, q_2) = (500 - q_1 - q_2) \), or, considering they have set a quantity that maximizes firm two’s price function, should they adopt firm two’s price function? If firm one did not lie, would the two firms be able to agree on a quantity?

(c) For question (b), what happens if firm one backs out of this agreement at the last moment? Suppose firm two agrees to the quantity from question (b) and when the two firms make their quantities and prices public, firm one declares their price to be \( (500 - q_1 - q_2) \). What quantity should firm one produce? What price will both firms charge?

(d) Using the same setup as in (a), if firm one declares the quantity they are going to produce and firm two picks a quantity in response to firm one, what quantity will firm one produce? What price will they charge?

(e) Using the same setup as in (a), if firm one declares the quantity they are going to produce and firm two picks a quantity in response to firm one, what quantity will firm two produce? What price will they charge?