Mixed ESSs in the context of Assignment 5

Finding the mixed ESS is quite easy, actually. For a two-strategy game with strategies $A$ and $B$, we define the strategy $C = pA + (1 - p)B$, where $p$ is the probability of playing $A$. This is an ESS provided that, if a newcomer arrives, he should expect the same payoff from playing either $A$ or $B$ against $C$,

$$\pi(A, C) = \pi(B, C).$$

Think of this in terms of our class playing rock-paper-scissors. If everyone is playing the ESS (1/3 each of rock, paper, and scissors), then it does not matter what a newcomer plays; they expect to win 1/3 of the time.

We can expand the above equation to yield,

$$\pi(A, C) = \pi(A, pA + (1 - p)B) = p\pi(A, A) + (1 - p)\pi(A, B),$$

and,

$$\pi(B, C) = \pi(B, pA + (1 - p)B) = p\pi(B, A) + (1 - p)\pi(B, B).$$

Equate this two as above. This readily solves for $p$:

$$p = \frac{\pi(B, B) - \pi(A, B)}{\pi(A, A) - \pi(B, A) - \pi(A, B) + \pi(B, B)}.$$

Consider assignment 4. We are given the payoff values, so we need only sub them into the above equation. Take (1a) for example. We are given,

<table>
<thead>
<tr>
<th></th>
<th>$A$</th>
<th>$B$</th>
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<tbody>
<tr>
<td>$A$</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>$B$</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

This yields,

$$p = \frac{2 - 1}{4 - 2 - 1 + 2} = \frac{1}{3}.$$