1. Given the following matrix game between players A and B,

\[
\begin{array}{cc|cc}
  & a_1 & a_2 \\
  b_1 & x, y & 2, 2 \\
  b_2 & 3, 1 & 3, 4 \\
\end{array}
\]

(a) what must be true of \(x\) and \(y\) for \((b_1, a_1)\) to be a strict Nash equilibrium?

(b) what must be true of \(x\) and \(y\) for there to be a mixed strategy Nash equilibrium where player B plays \(b_1\) with probability \(p^* = 3/4\) and player A plays \(a_1\) with probability \(q^* = 1/3\)?

(c) what are the expected payoffs of each player at the MSNE found in (b)?

2. Fill out the following matrix in such a way that it is an evolutionary game with a non-trivial MSNE (one in which each of the three strategies is played with non-zero probability) and have this MSNE also be an ESS:

\[
\begin{array}{ccc|ccc}
  & S_1 & S_2 & S_3 \\
  S_1 & & & \\
  S_2 & & & \\
  S_3 & & & \\
\end{array}
\]

3. Here’s a game we could have played in class. Suppose there are \(N\) of us. We each choose a positive integer (1, 2, 3, . . .). To win, a player must choose the lowest integer \(n\) such that fewer than \(n\) other players have selected \(n\). For example, if four people are playing, and one player chooses 1, two choose 2 and one chooses 3, the player that chose 1 has won and everyone else loses. If, on the other hand, two chose 1 and two choose 2, then the two who chose 2 have won.

(a) Find two different Nash equilibria for this game, for which all 145 you can win.

(b) Suppose every player uses a mixed strategy where they play \(n\) with probability \(p_n\). Write out an expression for the probability of winning in an \(N = 3\) play game as a function of the \(p_n\).

(c) Suppose \(N = 3\). Find all of the mixed strategy Nash equilibria.

4. Player one chooses a real number \(x\) and player 2 chooses a real number \(y\), both between 1 and 5. Player one’s payoff is

\[
\Pi_1(x, y) = -x^2 + 6x + y
\]

while player two receives

\[
\Pi_2(x, y) = -\frac{y^2}{x} + y.
\]

Find all Nash equilibria.
5. Find all the equilibria (pure NE and MSNE) in the following games

(a)

\[
\begin{array}{c|ccc}
& a_1 & a_2 & a_3 \\
\hline
b_1 & 2 & 1 & 0.2 \\
b_2 & 1 & -1 & -1 \\
b_3 & 3 & 0 & 1
\end{array}
\]

(b)

\[
\begin{array}{c|ccc}
& a_1 & a_2 & a_3 \\
\hline
b_1 & 0 & 0 & 1, -1 \\
b_2 & -1, 1 & 0 & 0 \\
b_3 & -1, 1 & 1 & 2
\end{array}
\]

6. Derive a system of replicator equations for the following games. Find the equilibria and check for stability.

(a)

\[
\begin{array}{c|cc}
S_1 & S_2 \\
\hline
S_1 & 2 & 1 \\
S_2 & 1 & 3
\end{array}
\]

(b)

\[
\begin{array}{c|ccc}
& S_1 & S_2 & S_3 \\
\hline
S_1 & 0 & -1 & 1 \\
S_2 & 1 & 0 & -1 \\
S_3 & -1 & 1 & 2
\end{array}
\]

7. You and your spouse have bought a box of 36 mint Girl Guide cookies. They are amazingly delicious but you notice that you experience diminishing marginal utility as you eat them. In fact, you calculate your utility to be \( u(x) = \ln(x) \), where \( x \) is the number of cookies you eat in a day. You both agree that what you do not eat today, you’ll share equally tomorrow. When you open the box you simultaneously commit to a number of cookies you will each eat today. If these two numbers sum to 30, then you both get 15.

(a) As a rational, utility-maximizing individual, how many cookies should you eat today? Tomorrow?
(b) If you both decide to maximize your total utility, not just your individual utility, how many should you eat today? Tomorrow?

8. Some researchers have suggested that the snowdrift game explains cooperation in humans more than the prisoner’s dilemma.

(a) Write down a payoff matrix for the snowdrift game. What are the (pure/mixed) evolutionary stable strategies?
(b) Suppose this game is played in a population at the equilibrium polymorphic state found in (a). We now change the game so that it is repeated \( N \) times each time two individuals get together. How does repetition change the polymorphism? Can you find a strategy that will invade this population? If so, what values of \( N \) permit this? If not, why not?

9. Two crows are fighting over a bagel outside the ARC. Each can choose their level of aggression, in terms of energy reserves, which is a parameter between 0 and 1. The more aggressive wins the bagel, which is worth 1 energy reserve point. If they are equally aggressive, they share the bagel. If both crows want to maximize their energy, what should they do?