Decoding

**Definition:** If \((F, S, E, D, P)\) is a channel coding model of length \(n\), then its *decoding error probability* is defined by

\[
P_{\text{err}} = \max_{c \in C} P_{\text{err}}(c),
\]
where \(C = E(S)\) is the set of codewords and

\[
P_{\text{err}}(c) = \sum_{\substack{y \in F^n \\ D(y) \neq c}} P(y|c).
\]

**Definition:** The *maximum likelihood decoder* (MLD) is defined by the rule

\[
D_{\text{MLD}}(y) = c \iff P(y|c) = \max_{c' \in C} P(y|c').
\]

**Note:** If, for a given \(y\), there are several \(c \in C\) which satisfy the RHS of (1), then we agree to take the smallest \(c\) wrt. a fixed total order on \(C\).

**Definition:** The *nearest neighbour decoder* (or nearest codeword decoder) is defined by the rule

\[
D_{\text{NCD}}(y) = c \iff d(y, c) = \min_{c' \in C} d(y, c'),
\]
where \(d\) is the Hamming distance.