**GRS Decoding: Method II**

**Euclidean Algorithm:** Given \( f, g \in F[X] \), put:

\[
\vec{v}_i = (r_i, s_i, t_i), \quad \text{for } i = -1, 0, \ldots, k
\]

where \( \vec{v}_{-1} = (f, 1, 0), \vec{v}_0 = (g, 0, 1) \) and

\[
\vec{v}_i = \vec{v}_{i-2} - q_i \vec{v}_{i-1} \text{ with } q_i = \text{quot}(r_{i-2}, r_{i-1}),
\]

when \( i \geq 1 \) and \( r_{i-1} \neq 0 \). If \( k \) is such that \( r_{k+1} = 0 \), then \( \gcd(f, g) = cr_k \), for some \( c \in F^\times \). Moreover, for \(-1 \leq i \leq k\), we have that

\[
(1) \quad s_i f + t_i g = r_i,
\]

\[
(2) \quad \deg(t_{i+1}) + \deg(r_i) = \deg(f).
\]

**Proposition 4:** In the situation of the key equation, apply the Euclidean algorithm to

\[
f(x) = x^{d-1} \quad \text{and} \quad g(x) = S(x).
\]

If \( h \leq k \) is chosen such that

\[
(3) \quad \deg(r_h) < \frac{d - 1}{2} \leq \deg(r_{h-1}),
\]

then \( c := t_h(0) \neq 0 \), and we have

\[
(4) \quad \Lambda(x) = t_h(x)/c \quad \text{and} \quad \Gamma(x) = r_h(x)/c.
\]