Problem 5(a): Test for pseudoprimes
The following program pseudo(n,b) checks whether n is a pseudoprime to the base b.
It returns true or false.

> pseudo := (n, b) → evalb(Power(b, n − 1, modp(n, 1)) mod n = 1);
Testing this with n := 30857, b = 3, b = 5 and b = 9:
> pseudo(30857, 3), pseudo(30857, 5), pseudo(30857, 9);
true, false, true

Thus, since n = 30857 is not a pseudoprime to the base 5, we see that n is composite.

(b) Test for Euler pseudoprimes
The following program Epseudo(n,b) checks whether n is an Euler pseudoprime to the base b.
It returns true or false.

> with(numtheory) :
> Epseudo := (n, b) → evalb(Power(b, (n − 1)/2, modp(n, numtheory:jacobi(b, n), n)) mod n = modp(numtheory:jacobi(b, n), n);

> Epseudo(30857, 3), Epseudo(30857, 5), Epseudo(30857, 9);
falsesh, falsesh, true

(c) The Solovay-Strassen primality test
> SolS := proc(n, k)
local i, r, r1, t;
    r := rand(1..n); t := true;
    for i to k while t do;
        r1 := r(
        t := Epseudo(n, r1);
    return(t)
end:
> n1 := 13999457;
> SolS(n1, 5), SolS(n1, 10), SolS(n1, 100);
false, falsesh, falsesh

Thus, n1 = 13999457 is composite, as already the first test told us. In fact,
n1 = 13999457 = 113*229*541.

> ifactor(n1);
(113) (229) (541)

> n2 := 104729;
> SolS(n2, 5), SolS(n2, 10), SolS(n2, 100);
true, true, true

Thus, we strongly suspect that n2 is prime. This is confirmed by Maple:
> isprime(n2);
true

> n3 := 340561;
Thus, already the first test tells us that $n^3$ is composite. In fact, $n^3 = 13\cdot17\cdot23\cdot67$ is a Carmichael number because it is squarefree and satisfies $(p-1)|(n^3-1)$ for all $p|n$:

```r
> ifactor(n^3);
(13) (17) (23) (67)
```

```r
> modp(n^3 - 1, 13 - 1), modp(n^3 - 1, 17 - 1), modp(n^3 - 1, 23 - 1), modp(n^3 - 1, 67 - 1);
0, 0, 0, 0
```

$SolS(n^3, 5), SolS(n^3, 10), SolS(n^3, 100);$