Realistic Time/Space Estimates
(for the Log Table Method)

Log Table Method: If $g \in \mathbb{F}_p^\times$ has order $n$, then this method requires (approximately):

1) Time: $n \log_2(p)$ bit operations

2) Space: $n \log_2(p)$ bits

Example: $p$ has 50 digits ($\approx 167$ bits), $n = p - 1$.

1) Time Analysis: A good computer might be able to compute $10 \text{ GHz} = 10^{10}$ bit operations per second. Thus, the log table for such a $g$ would take

$$10^{50}(50 \log_2(10))^2 \times 10^{-10} \approx 2.75 \times 10^{44} \text{ secs.}$$

Since 1 year has $60 \times 60 \times 24 \times 365 \approx 3.15 \times 10^7$ secs, this exceeds by far the present age of the universe:

15 billion years = $1.5 \times 10^{10}$ yrs $\approx 4.725 \times 10^{17}$ secs.

2) Space Analysis: The space required is

$$p \log_2(p) = 10^{50}(50) \log_2(10) \approx 1.66 \times 10^{52} \text{ bits.}$$

Since $100 \text{ TB} = 100 \times 10^{12} \times 8 = 8 \times 10^{14}$ bits, this far exceeds storage capacity of the whole world (7 billion people @ 100 TB storage:)

$$7 \times 10^9 \times 8 \times 10^{14} = 5.6 \times 10^{24} \text{ bits.}$$

Note: The supercomputer used by the German Climate Computing Centre (DKRZ) has 20 TB memory and 7,000 TB disk space and produces yearly 10,000 TB of data (Wikipedia).
**Remark:** The number of atoms in the universe is estimated to be

$$0.998 \times 10^{80} \approx 0.842 \times 2^{266}.$$ 

Thus, when $p$ has 100 digits, the log table of $\mathbb{F}_p^\times$ would require

$$p \log_2(p) = 10^{100}(100) \log_2(10) \approx 3.32 \times 10^{102}$$ bits,

which far exceeds the number of atoms in the universe!