Print your name and student number very clearly in the upper right
hand corner of your front sheet and staple your sheets if necessary.
Write in your own voice. Discussion with other students is encour-
gaged; but do not copy the work of others.

1) Let $f$ be the function on $[-\pi, \pi]$ given by $f(\theta) = \theta(\pi - \theta)$ for $0 \leq \theta \leq \pi$ and $f(\theta) = \theta(\pi + \theta)$ for $-\pi \leq \theta < 0$.

a) Sketch the graph of $f$.

b) How many continuous derivatives does $f$ have?

c) Find the Fourier series of $f$.

d) For which $\theta$ does the Fourier series of $f$ converge to $f(\theta)$.

2) Suppose we have two finite sequences $\{a_n\}_{n=1}^N$ and $\{b_n\}_{n=1}^N$ of com-
plex numbers. Let $B_0 = 0$ and $B_k = \sum_{n=1}^k b_n$ for $1 \leq k \leq n$. Show that
for $1 \leq M \leq N$

$$\sum_{n=M}^N a_n b_n = a_N B_N - a_M B_{M-1} - \sum_{n=M}^{N-1} (a_{n+1} - a_n) B_n.$$

3) Let $\{b_n\}_{n=1}^\infty \subset \mathbb{C}$ and $\{a_n\}_{n=1}^\infty \subset \mathbb{R}$ be two sequences such that

- $a_1 \geq a_2 \geq a_3 \geq \cdots \geq a_n \geq \cdots \geq 0$ and $\lim_{n \to \infty} a_n = 0$; and
- there is $M$ such that for all $N$ we have $|\sum_{n=1}^N b_n| \leq M$.

Show that the series $\sum_{n=1}^\infty a_n b_n$ converges.

4) Recall that in class we found the Fourier series of $f(\theta) = \theta$ on $[-\pi, \pi]$ to be

$$2 \sum_{n=1}^\infty (-1)^{n+1} \frac{\sin n \theta}{n}.$$ 

For which $\theta$ does this series converge? You don’t have to find the sum, just whether or nor it converges.

5) Suppose $\{f_k\}_{k=1}^\infty$ is a sequence of Riemann integrable functions on
$[-\pi, \pi]$ such that $\lim_{k \to \infty} \int_{-\pi}^{\pi} |f_k(\theta) - f(\theta)| \, d\theta = 0$. Show that $\hat{f}_k(n)$
converges uniformly to $\hat{f}(n)$. This means that for every $\epsilon > 0$ there is
$K$ such that for $k \geq K$ and all $n$ we have $|\hat{f}_k(n) - \hat{f}(n)| \leq \epsilon$. 

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