(1) Consider a DMS with alphabet $\mathcal{X} = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7\}$ and pmf described by

$$P_X(x) = \begin{cases} 
0.49 & \text{if } x = x_1 \\
0.26 & \text{if } x = x_2 \\
0.12 & \text{if } x = x_3 \\
0.04 & \text{if } x = x_4 \\
0.04 & \text{if } x = x_5 \\
0.03 & \text{if } x = x_6 \\
0.02 & \text{if } x = x_7 
\end{cases}$$

(a) Construct a (first-order) binary Huffman code for this source and compare its average code rate with the source entropy.

(b) Construct a (first-order) ternary Huffman code for this source and compare its average code rate with the source entropy (use appropriate units).

(2) Player A chooses some object in the universe, and player B attempts to identify the object with a series of yes-no questions. Suppose that player B is clever enough to use the code achieving the minimal expected length with respect to player A’s distribution. We observe that player B requires an average of 38.5 questions to determine the object. Find a rough lower bound to the number of objects in the universe.
Consider a suffix code, i.e., a code in which no codeword is a suffix of any other codeword. Show that such code is uniquely decodable, and show that the minimum average length over all (first-order) suffix codes for a DMS is the same as the average length of a (first-order) Huffman code designed for the same source.

Problem 3.1.

Problem 3.6.

Problem 3.16.

Consider the stationary Markov source described in Problem 7 of the previous assignment (HW # 2). If \( \rho = 1/2 \) and \( \delta = 3/2 \), design first-, second-, and third-order binary Huffman codes for this source. Determine in each case the average lengths per source sample (or average code rates) and compare them to the entropy rate.

Additional Problems for MATH 874 students:

Problem 3.12.

Problem 3.13.

Problem 3.21.