
MATH 892, WINTER 2010

COURSE OUTLINE

INSTRUCTOR: James A Mingo, mingo@mast.queensu.ca, Jeff 404, 3-2444

The course follows two books: Walter Rudin, *Real and Complex Analysis*, 3rd. ed. and Nicholas Young, *An Introduction to Hilbert Space*.

FROM RUDIN:

Chapter 10: Elementary Properties of Holomorphic Functions

§1 Complex differentiation, §2 Integration over paths, §3 The local Cauchy theorem, §4 The power series representation, §5 The open mapping theorem, §6 The global Cauchy theorem, §7 The calculus of residues.

Chapter 11: Harmonic Functions

§1 The Cauchy-Riemann equations.

Chapter 12: The Maximum Modulus Principle

§1 Introduction, §2 The Schwartz lemma.

Chapter 14: Conformal Mapping

§1 Preservation of angles, §2 Linear fractional transformations.

Chapter 18: Elementary Theory of Banach Algebras

§1 Introduction, §2 The invertible elements.

FROM YOUNG:

Chapter 7: Linear Operators

§1 The Banach space $\mathcal{L}(E, F)$, §2 Inverses, §3 Adjoints of operators, §4 Hermitian operators, §5 The spectrum.

Chapter 8: Compact Operators

§1 Hilbert-Schmidt operators, §2 The spectral theorem for compact Hermitian operators.

PRESENTATIONS Each student will be asked to make a twenty five minute presentation on a suitable topic. The grading scheme is as follows: assignments 40%, presentations 30%, final examination 30%.

ASSIGNMENT 1, DUE JANUARY 25

1) Rudin, Ch. 10, Exercise 3

2) Rudin, Ch. 10, Exercise 4

3) Rudin, Ch. 10, Exercise 6

4) Suppose f is a measurable function on \mathbb{R}^+ such that

- for each $b > 0$, $\int_0^b |f(t)| dt < \infty$;
- there are real numbers K, a, t_0 such that $K > 0$ and $t_0 \geq 0$, such that $|f(t)| \leq Ke^{at}$ for $t \geq t_0$.

Show that for each z with $\operatorname{Re}(z) \geq a$ the integral $\int_0^\infty f(t)e^{-zt} dt$ exists⁽¹⁾.

For $\operatorname{Re}(z) > a$, define $F(z) = \int_0^\infty f(t)e^{-zt} dt$. Show that F is analytic on $\operatorname{Re}(z) > a$.

5) In class we said that u is differentiable at (x_0, y_0) if there are constants A, B and functions ϵ and η defined on a neighbourhood of (x_0, y_0) such that

$$u(x, y) = u(x_0, y_0) + A(x - x_0) + B(y - y_0) + \epsilon(x, y)(x - x_0) + \eta(x, y)(y - y_0)$$

with $\lim_{(x,y) \rightarrow (x_0,y_0)} \epsilon(x, y) = \lim_{(x,y) \rightarrow (x_0,y_0)} \eta(x, y) = 0$. Show that this property is equivalent to: there exist constants A and B such that

$$\lim_{(x,y) \rightarrow (x_0,y_0)} \frac{u(x, y) - u(x_0, y_0) - A(x - x_0) - B(y - y_0)}{\sqrt{(x - x_0)^2 + (y - y_0)^2}} = 0$$

6) Prove equation (4) on page 199 of Rudin; that is for $n \geq 2$

$$\frac{z^n - w^n}{z - w} - nw^{n-1} = (z - w) \sum_{k=1}^{n-1} kz^{n-k-1} w^{k-1}$$

⁽¹⁾in the sense of Rudin Def. 1.30