1. (Exercise 1.35 p. 133 of Revuz & Yor) If $M$ is a continuous martingale and a Gaussian process (i.e. the finite-dimensional distributions are Gaussian), prove that $[M] = ([M]_t)_{t \in \mathbb{R}^+}$ is deterministic, i.e. there is a function $f$ on $\mathbb{R}^+$ such that $[M]_t = f(t)$ a.s. $\forall t \in \mathbb{R}^+$.

2. (Exercise 2.16 p. 143 of Revuz & Yor) Let $f$ be a locally bounded Borel function on $\mathbb{R}^+$ and $B = (B_t)_{t \in \mathbb{R}^+}$ a Brownian motion. Prove that the process $Z = (Z_t)_{t \in \mathbb{R}^+}$ given by
\[
Z_t = \int_{[0,t]} f dB, \quad \forall t \in \mathbb{R}^+, 
\]
is Gaussian and compute its covariance $\Gamma(s,t)$. Prove that $(\exp(Z_t - \frac{1}{2} \Gamma(t,t)))_{t \in \mathbb{R}^+}$ is a martingale. (Note: This generalizes the martingale construction $(\exp(B_t - \frac{1}{2}t))_{t \in \mathbb{R}^+}$ for Brownian motion.)

3. (Exercise 2.17 p. 143 of Revuz & Yor)
   (1) Let $B$ be a Brownian motion and $H$ an adapted right-continuous bounded process. Prove that for a fixed $t \in \mathbb{R}^+$
\[
\lim_{h \to 0} (B_{t+h} - B_t)^{-1} \int_{[t,t+h]} H dB = H_t \quad \text{in probability.}
\]
Show that the result is also true for $H$ unbounded if it is continuous. (Hint: One may apply Schwarz's inequality to
\[
E \left[ \frac{1}{B_t} \int_{[0,t]} (H - H_0) dB \right]^{1/2} 
\]
(2) Let $B = (B^1, \ldots, B^d)$ be a $d$-dimensional Brownian motion and for each $j$, let $H^j$ be a bounded right-continuous adapted process. Prove that for a fixed $t \in \mathbb{R}^+$,
\[
(B^1_{t+h} - B^1_t)^{-1} \sum_j \int_{[t,t+h]} H^j dB^j
\]
converges in law as $h \to 0$ to $H^1_t + \sum_{j=2}^d H^j_t (N^j/N1)$ where $(N^1, \ldots, N^d)$ is a centered Gaussian random variable with covariance $I_d$, independent of $(H^1_t, \ldots, H^d_t)$.

4. Let $X$ be a continuous adapted process. $X$ is a **continuous semimartingale** if there exists a continuous local martingale $M$ and a continuous adapted process $A$ of finite variation (with $A_0 = 0$ a.s.) such that $X = M + A$.

   (a) Show that the decomposition of a continuous semimartingale as a sum of a continuous local martingale and a continuous adapted process of finite variation is unique.

   (b) Let $X$ be a continuous semimartingale with decomposition $X = M + A$ as above. Let $t \in \mathbb{R}^+$, and let $(\pi^n_t)_{n \in \mathbb{N}^*}$ be a sequence of partitions of $[0,t]$ with $\delta \pi^n_t \to 0$ as $n \to \infty$. For each $n \in \mathbb{N}^*$, let
\[
S^n_t = \sum_{i} (X_{t_{i+1}}^{n} - X_{t_i}^{n})^2 
\]
where $\pi^n_t = (t_0^n, t_1^n, \cdots, t_n^n)$. Show that $S^n_t$ converges in probability to $[M]_t$ as $n \to \infty$. 