Math 110 Homework Assignment 3

DUE DATE: OCT. 7, 2016

1. List the different types of RREF possible for a 2×4 matrix (like we did in class for a 2×3 matrix).

BONUS QUESTION: How many different types of RREFs are there for a $2 \times n$ matrix?

2. Put the following matrices into RREF.

(a) $\begin{bmatrix} 3 & 2 & 1 \\ 6 & 5 & 4 \end{bmatrix}$

(b) $\begin{bmatrix} 10 & 6 & 9 \\ 15 & 9 & 12 \end{bmatrix}$

(c) $\begin{bmatrix} 2 & -6 & -1 & 6 \\ 4 & -12 & 2 & 20 \\ 3 & -9 & 0 & 12 \end{bmatrix}$

(d) $\begin{bmatrix} -1 & 6 & -1 & -1 \\ 3 & -18 & 1 & 0 \\ 2 & -12 & 3 & 0 \end{bmatrix}$

3. For each of the following matrices (already in RREF) parameterize all the solutions to the corresponding system of equations. Write your solution in vector form.

(a) $\left[\begin{array}{ccc|c} 1 & 0 & 3 & 8 \\ 0 & 1 & 2 & 2 \end{array}\right]$

(b) $\left[\begin{array}{cccc|c} 1 & -4 & 0 & -\frac{3}{2} & 5 \\ 0 & 0 & 1 & 6 & \frac{1}{9} \end{array}\right]$

(c) $\left[\begin{array}{ccccc|c} 1 & 0 & 3 & 0 & 1 & -5 \\ 0 & 1 & 2 & 0 & 8 & 9 \\ 0 & 0 & 0 & 1 & 7 & 4 \end{array}\right]$

(d) $\left[\begin{array}{cccccc|c} 1 & -2 & 0 & 3 & 0 & 11 & -7 \\ 0 & 0 & 1 & 4 & 0 & 6 & 21 \\ 0 & 0 & 0 & 0 & 1 & -9 & 14 \end{array}\right]$

4. Pigeons are sold at a rate of 5 for 3 coins, swans at a rate of 7 for 5 coins, and peacocks at the rate of 1 for 3 coins. A man was told to bring 100 birds costing 100 coins for the amusement of the King's son. How many of each kind does he buy? (This is a version of a problem from a 9th century Indian text.)

(a) If x is the number of pigeons, y the number of swans, and z the number of peacocks, write down the corresponding linear equations and the augmented matrix for the problem.

(b) Parameterize the solutions to the equations, where x, y, and z are real numbers.

(c) Finally, solve the problem of the riddle. Since x, y, and z are the numbers of birds, they must be integers, greater than or equal to zero. Also, according to the conditions of the problem, x must be a multiple of 5, and y a multiple of 7. There is more than one solution; find them all.