1. Put the following complex numbers in the form $a + bi$ (with $a, b \in \mathbb{R}$):

   (a) $(4 - 5i) - (6 - 3i)$  
   (b) $(2 - 3i) \cdot (4 + 5i)$  
   (c) $\frac{3 - 4i}{6 + 8i}$  
   (d) $(1 + i)^{20}$

2. Find all the solutions to $z^6 = 1$, where $z$ is a complex number. Draw the solutions on the unit circle in $\mathbb{C}$. Is there one solution whose powers give you all the rest? Is there more than one?

3. You can work out $\sin$ and $\cos$ of the angles $\pi/4$ and $\pi/3$ (or $\pi/6$) by looking at some special triangles, and most people know these values. But, there are actually more values that can be worked out exactly. In this question, let’s work out $\sin(2\pi/5)$ and $\cos(2\pi/5)$.

   To make the notation easier, let $c = \cos(2\pi/5)$ and $s = \sin(2\pi/5)$. Let $z = c + s \cdot i$.

   (a) Explain why $z^5 = 1$.

   (b) Expand $(c + s \cdot i)^5$ and collect the parts with $i$ (the imaginary part). Explain why this will be zero, so to find values of $c$ and $s$, we can start by looking for values where this polynomial in $c$ and $s$ is zero.

   In fact, since $s$ isn’t zero, and since the above expression is divisible by $s$, we can divide it by $s$ to get a degree four polynomial in $c$ and $s$.

   (c) Explain why $s^2 = 1 - c^2$, and substitute it into the polynomial above to get a polynomial of degree 4 which only involves the variable $c$.

   (d) If you look more closely, the polynomial from part (c) can also be considered as a polynomial in $c^2$, of degree 2 (i.e., you can write everything in terms of $c^2$). Use the quadratic formula to solve for $c^2$, and then take square roots to find the four possible solutions in $c$ to this polynomial.

   (e) To decide which of the four values above is really $\cos(2\pi/5)$, draw a picture of $z$ on the unit circle, and also draw a picture of the point at the angle $\pi/4$. From the picture, explain how you know that $0 < \cos(2\pi/5) < 1/\sqrt{2}$. There is only one solution in (d) which satisfies this condition, so that must be $\cos(2\pi/5)$. Which is it?

   (f) Use the formula $s^2 = 1 - c^2$ to find $\sin(2\pi/5)$.
4. Let $A$ be the matrix $\begin{bmatrix} 1 & 1 \\ -1 & 0 \end{bmatrix}$.

(a) Compute $A^2$, $A^3$, $A^4$, $A^5$, and $A^6$.

(b) What will $A^7$, $A^8$, and $A^9$ be?

(c) If we compute $A^k$ for $k = 1, 2, 3, 4, \ldots$, how many different matrices will we get?

(d) Find the characteristic polynomial of $A$, and compute its (complex!) eigenvalues.

(e) Looking at the answers to question (2), explain why $A$ has the behaviour above.

[Thinking about the fact that there is a formula for the entries of $A^k$ in terms of the eigenvalues of $A$, or what $A$ would look like in diagonal form is one way to think about (e), but there are others.]