DUE DATE: APR. 13, 2017

1. Let's start with the basis $w_1 = (5, 2)$ and $w_2 = (1, 12)$ for $\mathbb{R}^2$, and let w be the vector $w = (14, 23)$.

   (a) Write w as a linear combination of w_1 and w_2. The dot product rule doesn't work to find the coefficients here — why not?

   (b) Use the Gram-Schmidt process to find an orthogonal basis v_1, v_2 for $\mathbb{R}^2$ with $v_1 = w_1$.

   (c) Write w as a linear combination of v_1 and v_2.

   (d) Write v_1 and v_2 as linear combinations of w_1 and w_2.

   (e) Use the answers in (c) and (d) to recompute the answer for (a). Is this process easier than solving the equations?

2. Let $w_1 = (1, 2, 0, 3)$, $w_2 = (3, 5, 2, 5)$, $w_3 = (8, 1, 9, -8)$, and $w_4 = (6, 6, -9, 8)$, and consider the basis $\mathcal{B} = [w_1, w_2, w_3, w_4]$ for $\mathbb{R}^4$.

   (a) Use the Gram-Schmidt process to find an orthogonal basis v_1, v_2, v_3, and v_4 for $\mathbb{R}^4$.

Recall that as part of the process, not only will we get an orthogonal basis for $\mathbb{R}^4$, but for each $k = 1, 2, 3, 4$, the subspace spanned by $v_1, v_2, \ldots, v_k$ is the same as the subspace spanned by $w_1, w_2, \ldots, w_k$.

   (b) Show that the subspace spanned by v_2 and v_3 is *not* the same as the subspace spanned by w_2 and w_3.

   (c) Explain why (b) is not a contradiction with what the Gram-Schmidt process is supposed to do.

   (d) Let $v = (2, 41, -11, -14)$. Find the orthogonal projection of v onto the subspace spanned by w_1 and w_2.

NOTE: Your answer in (a) and (d) should consist of vectors whose entries are integers.

3. One important application of the idea of a "least squares solution" is trying to fit parameters in a model to data from an experiment. For instance, in linear regression, one looks for the best fit line. However, the method can be applied to fit the data to any type of function. This question is an illustration of that idea.

Here are three data points from a fictional experiment:

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$
y	1	$\sqrt{2}$	$\frac{5}{4}$

Suppose we want to approximate the data points by a formula of the form

$$y = a\cos(x) + b\sin(x).$$

(a) What are the linear equations that we wish a and b would satisfy?

(b) Find the least squares solution to these equations, and therefore an approximation to the data by a function of the form $a\cos(x) + b\sin(x)$.

NOTE: When solving a system of linear equations, it is fine to perform operations to simplify the equations (e.g. multiply by a scalar, take linear combinations). When setting up the least-squares approximation problem, operations like this can drastically change the answer. When answering part (b), use the equations that you found in (a) without changing them. (A small discussion of this issue will appear in the solutions.)

Note: This assignment is due on **Thursday, April 13**. You may hand the assignment in to my mailbox, in Jeff 507.