

1. Let

$$f(x) = \begin{cases} 3x + 2 & \text{if } x \neq 2 \\ 5 & \text{if } x = 2. \end{cases}$$

(a) Draw a sketch of the graph of f .

(b) Does $\lim_{x \rightarrow 2} f(x)$ exist? If so, what is its value?

In (b) you only have to answer yes or no (and give a value if you think the answer is yes).

(c) Give an ϵ - δ proof of your answer in (b).

That is, in (b), if you said that the limit does not exist, give an ϵ - δ proof that the limit does not exist (there have been such proofs in class). On the other hand, if you said that the limit does exist, and is equal to a particular number L , give an ϵ - δ proof that the limit is as claimed.

(d) Is f continuous at $x = 2$? If not, say which of the three conditions fail to hold.

2. In class we have looked at the round down function, $\lfloor \cdot \rfloor$. Similarly, there is the round up function $\lceil \cdot \rceil: \mathbb{R} \rightarrow \mathbb{Z}$, defined by

$$\lceil x \rceil = \min \left\{ n \in \mathbb{Z} \mid x \leq n \right\}.$$

For instance, $\lceil \pi \rceil = 4$, $\lceil -\pi \rceil = -3$, $\lceil e \rceil = 3$ and $\lceil 2 \rceil = 2$.

Let $g(x) = \lceil x \rceil \cdot \lfloor x \rfloor$. I.e., $g(x)$ is the product of the round-up of x and the round-down of x .

(a) Fill in the table with the value of $g(x)$ at the following values of x :

x	-1.5	-1.0	-0.5	0.0	0.5	1.0	1.5
$g(x)$							

(b) Does $\lim_{x \rightarrow 3^-} g(x)$ exist? If so, state the value you think it has. (Note that this is a one-sided limit.)

- (c) Does $\lim_{x \rightarrow 3^+} g(x)$ exist? If so, state the value you think it has.
- (d) Is g continuous at $x = 3$?
- (e) Is g continuous at $x = 0$?

In parts (b)–(e) you do not have to give an ϵ - δ argument. But, do explain your reasoning in each case. In particular, in parts (d) and (e), if you think g is not continuous there, say which of the three conditions fail to hold.

3. Let $a \in \mathbb{R}$ be a number, and define the function $h(x)$ by

$$h(x) = \begin{cases} \frac{x^2 + ax - 16}{2x - 4} & \text{for } x \neq 2 \\ 5 & \text{for } x = 2 \end{cases}$$

Note that the function h depends on the choice of the number a .

- (a) Find a value of a so that $\lim_{x \rightarrow 2} h(x)$ exists.

(HINT : Is there a value of a for which $x = 2$ is a root of $x^2 + ax - 16$? How is this related to the problem?)

- (b) With this value of a , is h continuous at $x = 2$?

4. Find $\lim_{x \rightarrow \infty} \frac{\sin(x)}{x^2}$, and justify your claim.