Math 120 — Homework Assignment 7

due date: Oct. 28, 2025

1. Interpret each of the following limits as computing the derivative $f'(c)$ of some function f at some point c, and then use the derivative function to evaluate the limit.

For instance, given the limit $\lim_{h\to 0} \frac{(3+h)^5 - 3^5}{h}$, we might say :

" Aha![1] If we choose the function $f(x) = x^5$, and $c = 3$, then the limit above is the limit computing $f'(c)$. Since we know that $f'(x) = 5x^4$, we know that $f'(c) = f'(3) = 5 \cdot 3^4 = 405$. Therefore the value of the limit above is 405."

(a) $\lim_{h\to 0} \dfrac{\frac{\sin(\pi/4+h)}{\cos(\pi/4+h)} - 1}{h}$.

(b) $\lim_{x\to 0} \dfrac{\sin(x) - 0}{x}$

(c) $\lim_{w\to \pi/2} \dfrac{\sin(w)\cos(w)}{w - \pi/2}$

(d) $\lim_{h\to 0} \dfrac{(4)^{7+h} - 4^7}{h}$.

2. Compute the derivative $f'(x)$ for each of the following functions $f(x)$. You do not have to simplify your answer.

(a) $f(x) = \sin((x+1)^2(x+2))$.

(b) $f(x) = \sin^2((x + \sin(x))^2)$.

(c) $f(x) = \sin(x\sin(x)) + \sin(\sin(x^2))$.

(d) $f(x) = \sin^2(x)\sin(x^2)\sin^2(x^2)$.

(e) $f(x) = (x + \sin^5(x))^6$.

(f) $f(x) = \sin((\sin^7(x^7) + 1)^7)$.

(g) $f(x) = \sin(x^2 + \sin(x^2 + \sin(x^2)))$.

(h) $f(x) = \dfrac{\sin(x^2)\sin^2(x)}{1 + \sin(x)}$.

[1] The use of "Aha!" in your answer is not required.

(i) $f(x) = \sin\left(\dfrac{x^3}{\sin\left(\frac{x^3}{\sin(x)}\right)}\right)$.

3. Why do we use radians in calculus?

Let $\sin(\theta)$, $\cos(\theta)$, and $\tan(\theta)$ be the usual trigonometric functions; they take their input in radians and output a number. Let SIN, COS, and TAN be the corresponding functions which take their input in degrees.

So, for example

$$\sin(\pi/2) = 1, \quad \cos(\pi) = -1, \qquad \text{and} \quad \tan(\pi/4) = 1$$

$$\text{corresponds to}$$

$$\mathrm{SIN}(90) = 1, \quad \mathrm{COS}(180) = -1, \quad \text{and} \quad \mathrm{TAN}(45) = 1,$$

but $\sin(90) = 0.893996664$, $\mathrm{COS}(\pi) = 0.99849715$, and $\tan(45) = 1.619775191$ (you can check using a calculator).

The relationship between these functions is $\mathrm{SIN}(\theta) = \sin(\frac{\pi}{180}\theta)$, $\mathrm{COS}(\theta) = \cos(\frac{\pi}{180}\theta)$, and $\mathrm{TAN}(\theta) = \tan(\frac{\pi}{180}\theta)$, since multiplying by $\frac{\pi}{180}$ is how we convert from degrees to radians.

(a) Find the derivatives of SIN, COS, and TAN. Write your answer in terms of SIN, COS, and TAN.

Suggestion for (a) : Chain rule!

(b) Find the derivative $\frac{d}{dx}\sin\Big(\sin\big(\cos(x) + 5x\big) + 4x^3\Big)$.

(c) Find the derivative $\frac{d}{dx}\mathrm{SIN}\Big(\mathrm{SIN}\big(\mathrm{COS}(x) + 5x\big) + 4x^3\Big)$.