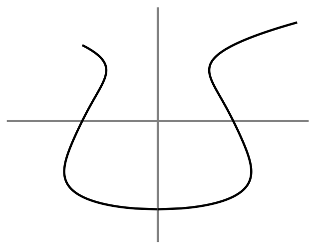
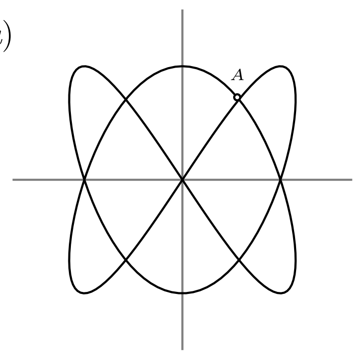
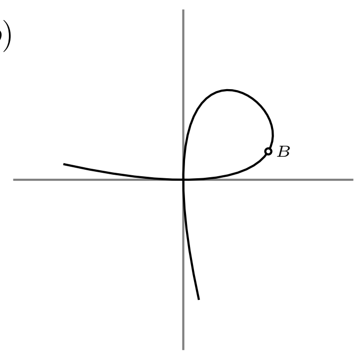
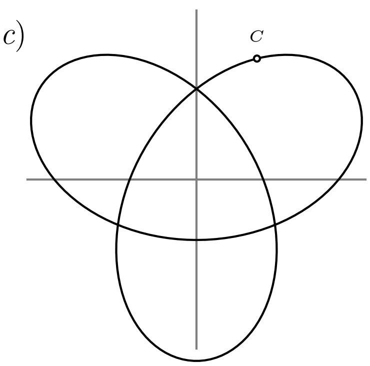
DUE DATE: Nov. 4, 2025

1. Here is a sketch of the curve given by the equation $y^5 - y - x^2 + 1 = 0$:



There are three different points on the curve where $x = 1$. Find the points and the equation of the tangent line through each point.

2. For each of the curves and points below, find the slope of the tangent line to the curve at the indicated point. (All the slopes, even in (a), are rational numbers.)

CURVE	(a) 	(b) 	(c) 
EQUATION	$16x^6 - 24x^4 + 9x^2 + 4y^4 - 4y^2 = 0$	$x^3 + 3x^2y + 3xy^2 + y^3 - 8xy = 0$	$4(x^2+y^2)^2 - 27(x^2+y^2-1)$ $-12x^2y + 4y^3 = 0$
POINT	$A = (\frac{\sqrt{15}}{8}, \frac{3\sqrt{15}}{16})$	$B = (\frac{9}{8}, \frac{3}{8})$	$C = (1, 2)$

REMARKS :

(1) The curve in (a) is an example of a *Lissajous curve*.

(2) The point A looks like it might be right at the spot where two pieces of the curve cross, but it is actually just above that. The point where the pieces cross is $(\frac{1}{2}, \frac{1}{\sqrt{2}})$.

(3) The curve in (b) is a variation on the Folium of Descartes, even though the equation looks a bit different.

(4) The curve in (c) is an example of a *trefoil*.

3. Find the derivatives of the following functions

(a) $3 \cdot e^{x^3} - 5 \cdot \cos(4^x)$.

(b) 2^{7^x} (this means $2^{(7^x)}$, and not $(2^7)^x$).

(c) $5^{\sqrt{x}+6^x} - 7\cos(8x \cdot e^{3x})$.

(d) $\cos(e^x) + e^{\cos(x)}$.

(e) $\cos(2^x) + 3^{\cos(x)}$.

4. Consistency check. When discovering a new mathematical fact, it is good to see how this fact interacts with things we already know, and in particular to check that the new fact is consistent with what we already know. Let us investigate how our formula for differentiating exponential functions interacts with our other rules for differentiation.

At the beginning of the classes on exponential functions, we realized that for each $b > 0$ there is a number, which we called a_b, such that $\frac{d}{dx} b^x = a_b \cdot b^x$.

At the end of those classes we found out a formula for a_b, but for most of this question let us go back to our first notation, writing (for example) $\frac{d}{dx} 13^x = a_{13} 13^x$, and similarly for other values of b.

(a) Find $\frac{d}{dx} 12^x$.

(b) Use the product rule to calculate $\frac{d}{dx} (3^x \cdot 4^x)$.

(c) Find $\frac{d}{dx} 8^x$.

(d) Use the chain rule to calculate $\frac{d}{dx} 2^{3x}$.

(e) Find $\frac{d}{dx} \left(\frac{1}{7}\right)^x$.

(f) Use the quotient rule to calculate $\frac{d}{dx} \frac{1}{7^x}$.

Now let us think about these calculations.

(g) Since $12^x = 3^x \cdot 4^x$, the answers in (a) and (b) must be equal. What identity does this imply between a_{12}, a_3 and a_4?

(h) Since $2^{3x} = (2^3)^x = 8^x$, the answers in (c) and (d) must be equal. What identity does this imply between a_8 and a_2?

(i) Since $\left(\frac{1}{7}\right)^x = \frac{1}{7^x}$, the answers in ($e$) and ($f$) must be equal. What identity does this imply between $a_{\frac{1}{7}}$ and a_7?

(j) Are these last three identities consistent with our formula $a_b = \ln(b)$?