DUE DATE: NOV. 18, 2025

1. Use the method of logarithmic differentiation to compute the following derivatives.

(a) $\frac{d}{dx}\left(3+\sin(x)\right)^{\left(e^{x^2}\right)}$.

(b) $\frac{d}{dx}\left(2^x+3x^4\right)^{(\ln x)}$.

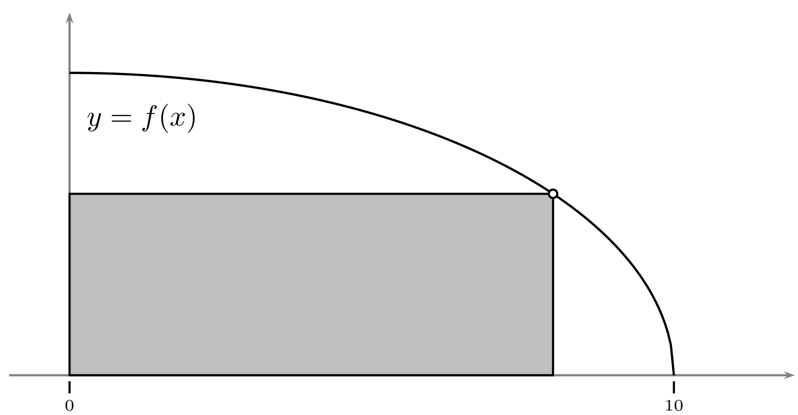
2. Find the critical points and the global maximum and minimum values of

$$f(x)=\frac{x-1}{x^2+15}$$

on the interval $[-9, 9]$.

3. Let $f(x)=\frac{1}{2}\sqrt{100-x^2}$; The graph of $f(x)$ is shown below. Find the area of the largest rectangle in the first quadrant with one corner on the graph, one corner at $(0,0)$ and sides parallel to the x- and y-axes.

(You may assume that the corner on the graph has its x-coordinate in $[0,10]$.)



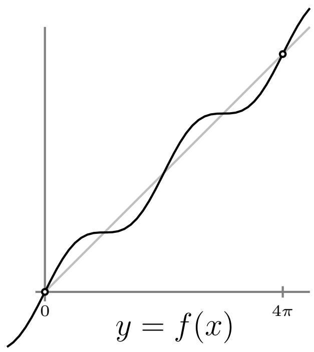
4. Let $f(x)=2\arctan(\sqrt{x})-\arcsin\left(\frac{x-1}{x+1}\right)$. The domain of f is $\{x\in\mathbb{R}\mid x\geqslant 0\}$.

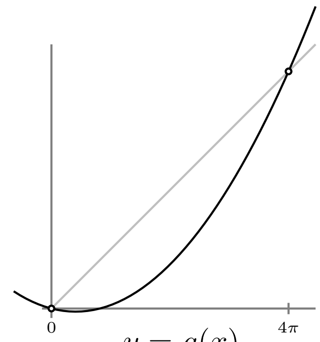
(a) Evaluate $f(0)$.

(b) Compute $f'(x)$ and simplify it as much as you can (if it doesn't look simple, keep simplifying!).

(c) Prove the identity $2\arctan(\sqrt{x})=\arcsin\left(\frac{x-1}{x+1}\right)+\frac{\pi}{2}$.

5. Let $f(x) = x + \sin(x)$, and $g(x) = x + \frac{1}{10}x(x - 4\pi)$. The graphs of f and g on the interval $[0, 4\pi]$ are shown below. Note that $f(0) = g(0)$, that $f(4\pi) = g(4\pi)$, and that both functions are differentiable on all of $\mathbb{R}$.





(a) Compute the average rate of change of f on the interval $[0, 4\pi]$.

(b) Compute the average rate of change of g on the interval $[0, 4\pi]$.

By the mean value theorem, there is at least one point $c \in (0, 4\pi)$ such that $f'(c)$ is equal to the average rate of change of f in (a).

(c) Find all the points $c \in (0, 4\pi)$ where $f'(c)$ is equal to your answer in (a). Draw those points, and the corresponding tangent lines to the graph of f, on the picture above.

(d) Similarly, find all the points $c \in (0, 4\pi)$ where $g'(c)$ is equal to your answer in (b), and add the corresponding points and tangent lines to the graph of g above.

(e) Are any of the points in (c) the same as the points in (d)?

Consider the following mathematical statement.

Proposition. Suppose that f and g are functions which are continuous on an interval $[a, b]$, and differentiable on (a, b), and in addition suppose that $f(a) = g(a)$ and that $f(b) = g(b)$. Then there is at least one point $c \in (a, b)$ such that $f'(c) = g'(c)$.

This statement is true. Here is an incorrect proof.

False Proof. Since $f(a) = g(a)$ and $f(b) = g(b)$, the average rates of change of f and g on $[a, b]$ are the same. Let m be this average rate of change. By the mean value theorem there is a point $c \in (a, b)$ so that $f'(c) = m$. Also by the mean value theorem there is a point $c \in (a, b)$ so that $g'(c) = m$. Therefore $f'(c) = m = g'(c)$, and so $f'(c) = g'(c)$. □

(f) Find and explain the error in this argument.

(g) Assume the conditions of the proposition, and let $h(x) = f(x) - g(x)$. Apply the MVT to h to give a correct proof of the proposition.