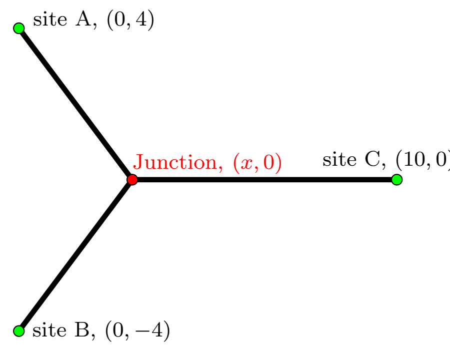
due date: Dec. 3, 2025

1. The illumination from a light source is inversely proportional to the square of the distance from that source. Two light sources are located 10 m apart. One light source is four times as bright as the other. Find the point between them where the illumination is the weakest.

Notes for problem 1: (*i*) As usual in max/min problems, you will have to first write down the the function you need to maximize. One way to start is to introduce a variable, say x, which is the distance from one of the sources of illumination. (*ii*) The resulting function will be undefined at the endpoints ($x = 0$ and $x = 10$ respectively). Besides finding the critical point, give an argument that that critical point is really an absolute min on the interval $(0, 10)$ (as opposed to being a local min, a local max, a global max, or any other possibility.)

2. In this problem we will determine the location for a Y-junction which minimizes the total distance to three sites.

Suppose the sites are at coordinates $(0, 4)$, $(0, -4)$, and $(10, 0)$. The symmetry implies that the junction should lie on the positive x-axis, but we must still determine the x-coordinate.



(*a*) Find the point $(x, 0)$ that minimizes the sum of the distances to the three sites.

(*b*) Determine the angle formed by the segments connecting the three sites to the optimal junction.

What happens if we move the location of the third site much closer to the first two?

(*c*) Suppose that the three sites are at $(0, 4)$, $(0, -4)$, and $(2, 0)$ and find the point $(x, 0)$ that is the minimum total distance from these three sites.

3. Go through steps **A**–**G** in class for the function xe^{-x^2}, and then sketch the graph of the function. (I.e., step **H**.)

As a reminder, here are the steps.

A: Domain; **B:** Intercepts; **C:** Symmetries; **D:** Asymptotes; **E:** Intervals where the function is increasing/decreasing; **F:** Critical points, and their classification; **G:** Intervals where the function is convex/concave; **H:** Sketch the graph.

4. Consider the function $f(x) = axe^{-bx}$ where a and b are positive constants.

(a) Find the local maxima, local minima, and points of inflection.

(b) How does varying a and b affect the shape of the graph?

(c) On the same set of axes, sketch the graph of the function for several values of a and b (three should be enough).

(d) If we want the maximum of the function to occur at $(2, 3)$, what must the values of a and b be?

Note: It often happens that we know from theoretical considerations that some physical process can be described by a function of a certain form with unknown parameters (like axe^{-bx} above). By performing experiments we can hope to find enough data points to determine the parameters. Because there is always experimental error, it is easier (and more accurate) to fit the parameters by looking for clearly defined features of the results (local maxes or mins, inflection points) and adjusting the parameters so that these features occur at the points observed by experiment (i.e., some version of part (d) above).

Note: This homework assignment is due on **Wednesday, December 3rd**, by 22:00. (I.e., one day later than the usual Tuesday deadline.)