1. Let $F(x, y) : \mathbb{R}^2 \rightarrow \mathbb{R}^3$ be the function $F(x, y) = (e^{x^2}, y\sin(\pi x), xy)$, and $G : \mathbb{R}^3 \rightarrow \mathbb{R}^2$ be the function $G(u, v, w) = (\cos(uv), w - u^2)$.

(a) Compute $F(1, 1)$, and let $q$ be this point in $\mathbb{R}^3$.

(b) Write out the composite function $G \circ F$, and compute directly $D(G \circ F)(1, 1)$.

(c) Compute $DF(1, 1)$ and $DG(q)$.

(d) Compute the product $DG(q) \cdot DF(1, 1)$ and verify the chain rule in this case.

2. Let $F : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be the function $F(r, \theta) = (r\cos(\theta), r\sin(\theta))$. Note that $F(2, \frac{\pi}{3}) = (1, \sqrt{3})$.

(a) Compute $DF(2, \frac{\pi}{3})$.

Suppose that $G : \mathbb{R}^2 \rightarrow \mathbb{R}$ is a differentiable function, and define a new function $H$ by $H = G \circ F$.

(b) What formula does the chain rule give for computing $DH(2, \frac{\pi}{3})$ in terms of $DG$ and $DF$?

(c) Suppose we know that $\frac{\partial H}{\partial r}(2, \frac{\pi}{3}) = 2$ and $\frac{\partial H}{\partial \theta}(2, \frac{\pi}{3}) = 4$. Use your answer from (b) to find $\frac{\partial G}{\partial x}(1, \sqrt{3})$ and $\frac{\partial G}{\partial y}(1, \sqrt{3})$. (This will involve inverting a matrix.)

(d) Suppose we are at the point $(1, \sqrt{3})$. In what direction $\vec{v}$ should we go so that the instantaneous rate of change of $G$ through $(1, \sqrt{3})$ in the direction $\vec{v}$ is zero?

3. 

(a) Describe and sketch the graph $z = \frac{1}{x^2 + y^2}$.

(b) Show that the parameterization $(x(t), y(t), z(t)) = (e^t \cos(t), e^t \sin(t), e^{-2t})$. lies on the graph from part (a).

(c) Describe what this curve does, and sketch it on the graph from part (a).