Math 280 Homework Assignment 6

DUE DATE: OCT. 26, 2017

1. If f is a function $f\colon \mathbb{R}^3 \longrightarrow \mathbb{R}$, and $\mathbf{F}$ a vector field on $\mathbb{R}^3$, prove that

$$\operatorname{Div}(f\mathbf{F}) = f\operatorname{Div}(\mathbf{F}) + \mathbf{F}\cdot\operatorname{grad}(f).$$

2. Find a parameterization of the curve $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 1$. Is your parameterization continuous? Differentiable? Piecewise C^1? C^1?

3. Compute the integral of $f(x,y) = xy - x - y + 1$ along the following curves connecting the points $(1,0)$ and $(0,1)$.

(a) $\mathbf{c}_1$: the circular arc $\mathbf{c}_1(t) = (\cos(t),\, \sin(t))$, $0 \leqslant t \leqslant \pi/2$.

(b) $\mathbf{c}_2$: the straight line segment $\mathbf{c}_2(t) = (1-t, t)$, $0 \leqslant t \leqslant 1$.

(c) $\mathbf{c}_3$: the path from $(1,0)$ horizontally to the origin, then vertically to $(0,1)$.

(d) $\mathbf{c}_4$: from $(1,0)$ vertically to $(1,1)$, then horizontally to $(0,1)$.

(e) $\mathbf{c}_5$: the circular arc $\mathbf{c}_5(t) = (\cos(t),\, -\sin(t))$, $0 \leqslant t \leqslant 3\pi/2$.

4. Find the integral $\int_{\mathbf{c}} f\, ds$ where $f(x,y,z) = \sqrt{9xz + 4y + 1}$ and $\mathbf{c}$ is the "twisted cubic" : $\mathbf{c}(t) = (t, t^2, t^3)$ with $t \in [0,4]$.

5. Find the average value of the function $f(x,y,z) = xyz$ along the helix $\mathbf{c}(t) = (\sin(t), 8t, \cos(t))$, $t \in [0, 6\pi]$.

NOTE: The average value of a function f over an interval $[a,b]$ is the integral of f over $[a,b]$ divided by the length of $[a,b]$. Similarly, the average value of a function f over a curve $\mathbf{c}$ is the integral of f over $\mathbf{c}$, divided by the length of $\mathbf{c}$.