1. Which of the following sets are path connected? Which are simply connected?
   (a) $\mathbb{R}^2$ with the circle $x^2 + y^2 = 1$ removed.
   (b) $\mathbb{R}^3$ with the circle $x^2 + y^2 = 1$, $z = 0$ removed.
   (c) The set $\{(x, y) \mid 1 < x^2 + y^2 < 2\}$ in $\mathbb{R}^2$.
   (d) $\mathbb{R}^3$ with the helix $(\cos(t), \sin(t), t)$, $t \in [0, \pi]$ removed.
   (e) The set $\{(x, y) \mid x^2 - y^2 < 0\}$ in $\mathbb{R}^2$.

2. Here are three curves connecting the point $(1, 0, 0)$ to the point $(-1, 0, 0)$ in $\mathbb{R}^3$:
   $c_1$: The half-circle $(\cos(t), \sin(t), 0)$, $t \in [0, \pi]$.
   $c_2$: The segment $(-t, t^2 - 1, 1 - t^2)$ of a parabola, $t \in [-1, 1]$.
   $c_3$: The straight line $(-t, 0, 0)$, $t \in [-1, 1]$.
   (a) For $F = (-y, x, z)$, compute $\int_{c_1} F \cdot ds$, $\int_{c_2} F \cdot ds$, and $\int_{c_3} F \cdot ds$.
   (b) For $G = (e^{yz}, xz e^{yz}, xy e^{yz})$, compute $\int_{c_1} G \cdot ds$, $\int_{c_2} G \cdot ds$, and $\int_{c_3} G \cdot ds$.
   (c) Is $F$ a conservative vector field? Is $G$?

3. Let $F$ be the vector field
   \[ F(x, y) = \left( \frac{y}{x^2 + y^2}, \frac{-x}{x^2 + y^2} \right) \]
   and $c$ the unit circle, oriented counterclockwise.
   (a) What is the domain of definition of the vector field $F$? Is it simply connected?
   (b) Compute $\text{Curl}(F)$ (the \(\mathbb{R}^2\) curl, which is a scalar function, and not a vector field).
   (c) Compute $\int_c F \cdot ds$.
   (d) If $G$ is a vector field, and $G = \nabla g$ for some function $g$, what would $\int_c G \cdot ds$ have to be? (Hint: Think of $c$ as a curve whose ending point is the same as its starting point).
   (e) Explain how you know that $F$ cannot be the gradient of any function, even though by a local calculation (the curl) it looks like it should be.
4. Let $f$ be the function $f(x, y) = x^2 y$, and

$c_1$: The half circle $(\sqrt{2} \cos(t), \sqrt{2} \sin(t)), t \in [-3\pi/4, \pi/4]$.
$c_2$: The half circle $(\sqrt{2} \cos(t), -\sqrt{2} \sin(t)), t \in [3\pi/4, 7\pi/4]$.
$c_3$: The straight line $(t, t), t \in [-1, 1]$.

All three curves connect the point $(-1, -1)$ to the point $(1, 1)$.

(a) compute $f(1, 1) - f(-1, -1)$
(b) Let $\mathbf{F} = \nabla f$. Compute $\mathbf{F}$.
(c) Compute $\int_{c_1} \mathbf{F} \cdot ds$, $\int_{c_2} \mathbf{F} \cdot ds$, and $\int_{c_3} \mathbf{F} \cdot ds$.
(d) Explain the connection between (a) and (c).