1. For each of the following regions R, first sketch the region, and then set up the integral $\iint_R f(x,y)\,dA$ as a type I integral and a type II integral. (You do not have to evaluate the integrals, and in fact, there is no explicit function given to integrate.)

(a) The region $R = \{(x,y) \in \mathbb{R}^2 \mid x^2+y^2 \leqslant 1,\ y \geqslant 0\}$.

(b) The region $R = \left\{(x,y) \in \mathbb{R}^2 \,\middle|\, 0 \leqslant x \leqslant \frac{\pi}{2},\ 0 \leqslant y \leqslant \sin(x)\right\}$.

(c) The region $R = \{(x,y) \in \mathbb{R}^2 \mid -2 \leqslant x \leqslant 2,\ |x| \leqslant y \leqslant 2\}$.

(d) The region $R = \{(x,y) \in \mathbb{R}^2 \mid 3 \leqslant x \leqslant 5,\ x^2 - y^2 \geqslant 9\}$.

2. Find $\iint_R f(x,y)\,dA$ in each of these cases:

(a) $f(x,y) = \cos(x+y)$, $R = [0, \frac{\pi}{2}] \times [0, \frac{\pi}{4}]$

(b) $f(x,y) = \frac{y^2}{\sqrt{1+x^2}}$, $R = [0,5] \times [1,4]$.

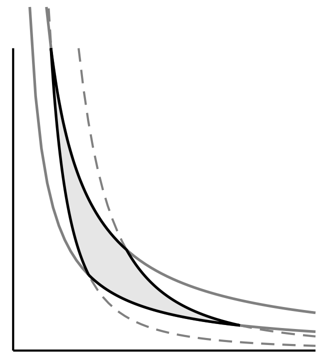
(c) $f(x,y) = x^2$, R is the triangular region bounded by the lines $x = 4$, $y = 3$, and $x + y = 4$.

3. For each of the following integrals, change the order of integration and then compute the integral.

Reminder: There is no magic formula to know how to change the order of the integration. The only thing to do is to use the given iterated integral to sketch the region of integration, and then use the sketch to reverse the order.

(a) $\displaystyle\int_0^4 \left(\int_{\frac{y}{2}}^2 e^{x^2}\,dx \right) dy$ (b) $\displaystyle\int_0^1 \left(\int_0^{\arccos(y)} e^{\sin(x)}\,dx \right) dy$ (c) $\displaystyle\int_{-1}^1 \left(\int_0^{\sqrt{1-x^2}} x\cos(y^2) + 6x^2 y\,dy \right) dx$

(d) $\displaystyle\int_{1/2}^1 \left(\int_1^{2y} \frac{\ln x}{x}\,dx \right) dy + \int_1^2 \left(\int_y^2 \frac{\ln x}{x}\,dx \right) dy$



4. In the picture at right, the solid grey lines are the curves $xy = 1$ and $xy = 2$, while the dashed grey lines are the curves $x^2y = 1$ and $x^2y = 3$. Let R be the shaded region between these curves.

The purpose of this question is to compute $\iint_R f(x,y)\,dA$ where $f(x,y) = x^2y^2$.

(a) Find parameterizations $x(u,v)$ and $y(u,v)$ in terms of u and v so that $x^2y = u$ and $xy = v$.

(b) In terms of the u, v parameterization, what are the limits of integration?

(c) What is the function f in terms of u and v?

(d) Compute the Jacobian of this parameterization.

(e) Write down and compute the integral in terms of the u and v parameterization. (i.e., use the change of variables theorem to compute the integral above.)