1. This problem is related to the change of variables example from class.

(a) For any two positive numbers $a$ and $b$ show, either geometrically or algebraically, that there is only one solution $(x, y)$ in the positive quadrant to $xy = a$ and $y = bx$.

(b) Part (a) shows that the functions $xy$ and $y/x$ are good coordinates on the positive quadrant – they’re sufficient to distinguish any two points in that quadrant. If $xy = u$ and $y/x = v$ are these new coordinates, find formulas for $x$ and $y$ in terms of $u$ and $v$.

(c) Sketch the region $R$ in the $xy$-plane bounded by $xy = 1$, $xy = 3$, $y = x$ and $y = 4x$. How would you describe $R$ in terms of $u$, $v$ coordinates?

(d) Compute the Jacobian of the change of coordinates (the determinant of the derivative matrix).

(e) If $f(x, y)$ is the function $f(x, y) = x^3y^7$, find the integral $\int \int_R f(x, y) \, dA$ by changing to $u$, $v$ coordinates.

2. In the picture at right, the solid grey lines are the curves $xy = 1$ and $xy = 2$, while the dashed grey lines are the curves $x^2y = 1$ and $x^2y = 3$. Let $R$ be the shaded region between these curves.

The purpose of this question is to compute $\int \int_R f(x, y) \, dA$ where $f(x, y) = x^2y^2$.

(a) Find parameterizations $x(u, v)$ and $y(u, v)$ in terms of $u$ and $v$ so that $x^2y = u$ and $xy = v$.

(b) In terms of the $u$, $v$ parameterization, what are the limits of integration?

(c) What is the function $f$ in terms of $u$ and $v$?

(d) Compute the Jacobian of this parameterization.

(e) Write down and compute the integral in terms of the $u$ and $v$ parameterization. (i.e., use the change of variables theorem to compute the integral above.)
3. Describe the volume being integrated over, and compute the iterated integral

(a) \[ \int_{-\sqrt{8}}^{\sqrt{8}} \int_{-\sqrt{8-x^2}}^{\sqrt{8-x^2}} \int_{-3}^{8-x^2-y^2} 2 \, dz \, dy \, dx \]

(b) \[ \int_{0}^{1} \int_{0}^{\sqrt{1-y^2}} \int_{0}^{\sqrt{1-x^2-y^2}} 2z \, dz \, dx \, dy \]

4. Sketch the region of integration for the iterated integral \( \int_{0}^{1} \int_{0}^{y} \int_{0}^{x} x^2yz \, dz \, dx \, dy \), and express it in the five other possible orders of integration.

(You do not have to evaluate any of the integrals.)

5. For each of the following surfaces, find a parameterization, and compute the tangent vectors and normal vectors in terms of that parameterization:

(a) The graph of \( f(x, y) = 9 - x^2 - y^2 \) over the points where the function is positive.

(b) The part of the paraboloid \( z = x^2 + y^2 \) in the first octant.

(c) The surface obtained by rotating the circle \((y - 3)^2 + z^2 = 1, \ x = 0\) about the \(z\)-axis.