1. Compute the integral \( \iint_S f \, dS \) for the function \( f(x, y, z) = xy \) and the surface \( S \) which is the graph of \( z = x^2 + y^2 \) inside the rectangle \( 0 \leq x \leq 3, \ 0 \leq y \leq 2 \).

2. Fix a radius \( r > 0 \) and two angles \( \varphi_1 \) and \( \varphi_2 \), with \( -\frac{\pi}{2} \leq \varphi_1 \leq \varphi_2 \leq \frac{\pi}{2} \). Find the surface area of the portion of the sphere of radius \( r \) with latitudes between \( \varphi_1 \) and \( \varphi_2 \).

3. Find the integral \( \iint_S \mathbf{F} \cdot dS \) where \( S \) is the helicoid parameterized by \( (u \cos(v), \ u \sin(v), \ v) \), \( 0 \leq u \leq 1, \ 0 \leq v \leq 4\pi \) with positive orientation upwards, and where \( \mathbf{F} \) is the vector field \( \mathbf{F}(x, y, z) = (y, -x, xz) \).

4. Find the flux integral of \( \mathbf{F}(x, y, z) = (z, x, y^2) \) through the top half of the unit sphere, with outward orientation.

5. Compute the flux integral of

\[
\mathbf{F}(x, y, z) = \left( \frac{x}{(x^2 + y^2 + z^2)^{3/2}}, \frac{y}{(x^2 + y^2 + z^2)^{3/2}}, \frac{z}{(x^2 + y^2 + z^2)^{3/2}} \right)
\]

through the sphere of radius \( r \), oriented outwards.

Compute the divergence \( \text{Div}(\mathbf{F}) \) of \( \mathbf{F} \). Don’t these two answers contradict the divergence theorem? Can you resolve this conflict?