1. Find the integral \( \iint_S \mathbf{F} \cdot dS \) where \( S \) is the helicoid parameterized by \((u \cos(v), u \sin(v), v)\), \(0 \leq u \leq 3, 0 \leq v \leq 4\pi\) with positive orientation upwards, and where \( \mathbf{F} \) is the vector field \( \mathbf{F}(x, y, z) = (xz, -yz, xy) \).

2. Find the flux integral of \( \mathbf{F}(x, y, z) = (x^2, y^2, z^2) \) through the top half of the unit sphere, with outward orientation.

3. The parameterized curve \( \mathbf{c}(t) = \left(5 \cos(t) + \sin(5t), 5 \sin(t) + \cos(5t)\right) \) for \( t \in [0, 2\pi] \) is shown at right. Use the vector field \( \mathbf{F} = \frac{1}{2}(-y, x) \) and Green’s theorem to find the area enclosed by the curve.

The angle addition formula \( \sin(\alpha+\beta) = \sin(\alpha) \cos(\beta)+\cos(\alpha) \sin(\beta) \) may prove useful at some point in the calculation.

4. Compute the following line integrals by using Green’s Theorem to convert each of them into an integral over a two-dimensional region \( R \), and then evaluating that integral over \( R \).

   (a) Compute \( \int_c \mathbf{F} \cdot ds \), where \( c \) is the circle of radius 2, centered at \((0,0)\), oriented counterclockwise, and \( \mathbf{F}(x, y) = \left(\cos(\cos(x)) - x^2 y, e^{\sin(y^2)} + xy^2\right) \).

   (b) Compute \( \int_c \mathbf{F} \cdot ds \), where \( c \) is the boundary of \([1, 2] \times [-1, 1]\), oriented counterclockwise, and \( \mathbf{F}(x, y) = \left(xy^2 + x^3, e^{x^2} + e^{y^2}\right) \).

   (c) Compute \( \int_c \mathbf{F} \cdot ds \), where \( c \) is the boundary of the region between \( y = x^2 - 4x \) and \( y = 5 \), oriented counterclockwise, and \( \mathbf{F}(x, y) = \left(y, x^2 y\right) \).
5. Consider the following integral, which does not seem very easy to evaluate.

\[ (*) \quad \frac{1}{\pi} \int_0^{2\pi} e^{100\cos^2(t)} \sin(1 + e^{30\cos^2(t)}) \sin(t) + \cos^2(t) \, dt \]

In this problem we will evaluate the integral by using Green’s theorem. Let \( c \) be the circle of radius 1 centered at (0, 0), and oriented counterclockwise. One possible parameterization of \( c \) is \( c(t) = (\cos(t), \sin(t)) \) with \( t \in [0, 2\pi] \).

(a) Find a vector field \( \mathbf{F} \) so that when evaluating \( \int_c \mathbf{F} \cdot ds \) using the parameterization above, the integral that results is \( (*) \).

(b) Use Green’s theorem to convert this to an integral over the unit disc, and evaluate that integral.