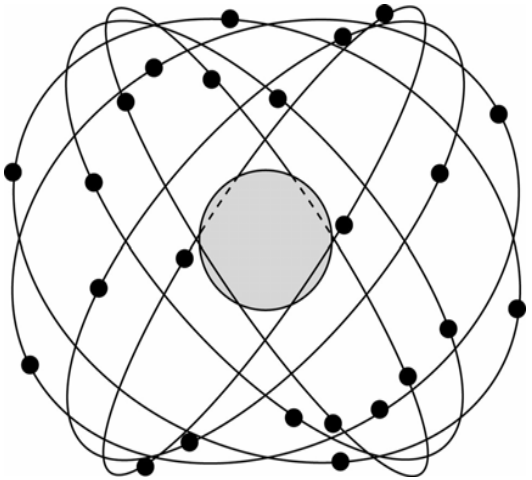
Global Positioning System (GPS)

Math 437/837

CISC490/850

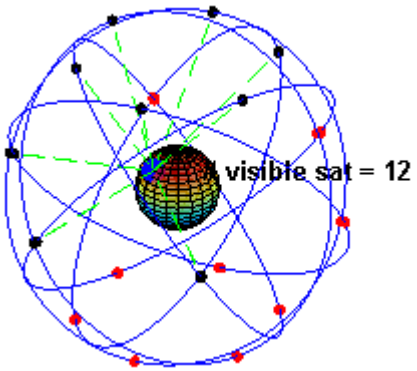


Satellites and Receivers

- Each satellite sends signals indicating its position relative to the receiver.

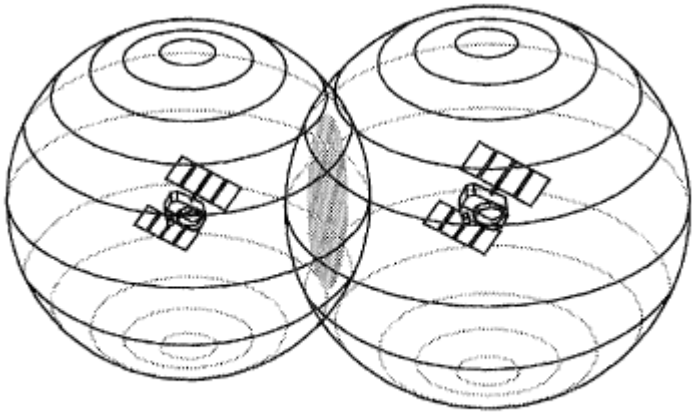
Satellites in orbit

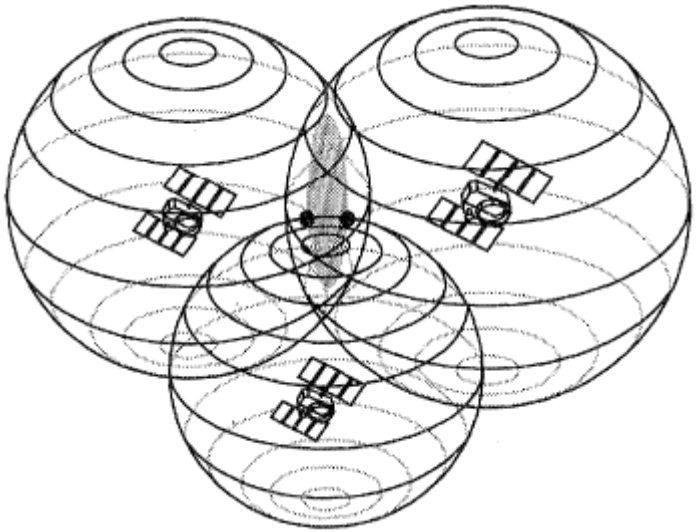
- This is an animation of 24 GPS satellites with 4 satellites in each of 6 orbits. It shows how many satellites are visible at any given time. This ensures redundancy to ensure accuracy.



The mathematics of GPS

- The intersection of two spheres is either empty or a circle.
- The circle will intersect a third sphere in at most two points.
- This geometric fact is the basis of GPS since other factors can be used to eliminate one of the two points as being an irrelevant solution to the problem.





Satellites and signals

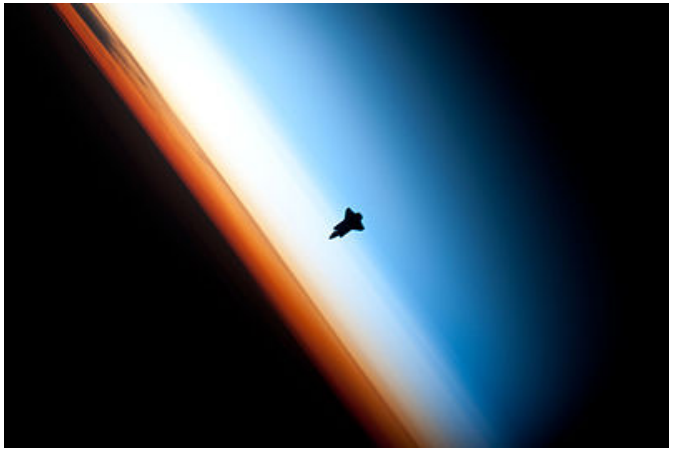
- Each satellite of the network sends a signal indicating its position and the time of the transmission of the signal.
- Since signals travel at the speed of light, the receiver can determine the radial distance of the satellite from the receiver.
- Many think that the receivers transmit information to the satellites, whereas in reality, it is the other way around.
- The receiver then uses basic math to determine its position.

Spheres

- If the receiver is R units away from satellite A, then the receiver lies on a sphere of radius R centered at A.
- A suitably positioned second satellite B can be used to determine another sphere, and the intersection of these two spheres determines a circle.
- A third satellite can be used to narrow the position to two points, and finally, a fourth, not coplanar with the other three, can be used to pinpoint the position of the receiver.

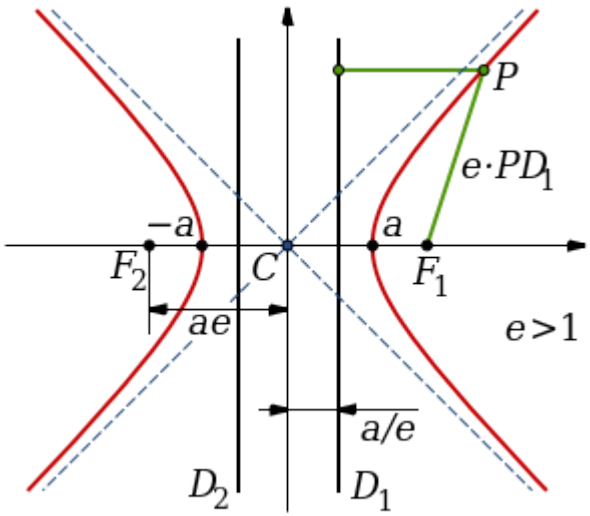
GPS errors

- Although light travels at 186,000 miles per second, its path is impeded by various factors such as the ionosphere and the troposphere.
- There are also errors due to the time mechanism.
- The satellites are equipped with atomic clocks and so their timing is very accurate.
- The receivers do not have accurate clocks and so this also introduces errors into the calculation.



Hyperbolas and errors

- Fix two foci in the plane. A hyperbola is the locus of points in the plane whose absolute value of the difference of the distances to the two foci is constant.
- Each receiver computes the radial distance to the satellite, but this calculation has errors in it.
- If the distances are P and Q, then the difference P-Q has no error in it. The receiver then lies on the hyperbola of revolution with the two satellites as the foci.
- Changing satellites gives another hyperboloid whose intersection with the first hyperboloid is a curve. A fourth satellite will then give the exact position without any error.



Other errors

- Signal transmission errors and noisy channels.
- Relativistic errors due to gravitational fields.
- Signal delays due to atmospheric disturbance.