MATH 437/837 (CISC 490/850): Assignment 1

Due: 29 January 2015

Undergraduates: do any eight questions.

Graduate students: do all ten questions.

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1. Calculate the determinant:

$$\begin{vmatrix} 0 & 0 & 0 & a & b \\ 0 & 0 & 0 & c & d \\ 0 & 0 & 0 & e & f \\ p & q & r & s & t \\ v & w & x & y & z \end{vmatrix}$$

2. Using Cramer's rule solve for x, y, z:

$$\begin{array}{rcrcrcr} x & + & 2y & + & z & = & 3 \\ 2x & + & 4y & + & z & = & 3 \\ 3x & + & 7y & & & = & 2 \end{array}$$

3. Show that three vectors in $\mathbb{R}^3$ given by $(x_1, y_1, z_1), (x_2, y_2, z_2)$ and (x_3, y_3, z_3) with respect to the standard basis are linearly dependent if and only if

$$\begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix} = 0.$$

   In particular, if the three points are collinear, show that the determinant is zero.

4. Let G be the set of $n \times n$ matrices A with real entries with the property that the sum of the entries of each row is 1. Show that G is **closed** under matrix multiplication.

5. Show that the product of two orthogonal matrices A_1 and A_2 with determinant 1 is itself an orthogonal matrix of determinant 1.

6. If A is an $n \times n$ matrix, show that A and A^t (the transpose of A) have the same determinant.

7. (a) Find the eigenvalues of the matrix A given by

$$\begin{pmatrix} 2 & 2 & 1 \\ 2 & 5 & 2 \\ 1 & 2 & 2 \end{pmatrix}$$

   (b) For each eigenvalue in (a), write down a basis for the corresponding eigenspace.

8. Show that the matrix

$$\begin{pmatrix} a+b & b-a \\ a-b & b+a \end{pmatrix}$$

   is orthogonal if and only if $a^2 + b^2 = 1/2$.

9. Show that for every real number a, the matrix

$$\frac{1}{2a^2+1}\begin{pmatrix} 1 & -2a & 2a^2 \\ 2a & 1-2a^2 & -2a \\ 2a^2 & 2a & 1 \end{pmatrix}$$

   is orthogonal.

10. A trained mouse lives in the house shown below. A bell rings at regular intervals and the mouse is trained to change rooms each time it rings. When it changes rooms, it is equally likely to pass through any of the doors in the room it is in. Approximately what fraction of its life will it spend in each room?