MATH 498/812: Assignment 4

Due: 22 November 2012

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1. Let $||x|| = \min\{|x - n| : n \in \mathbb{Z}\}$. Show that

$$||x + y|| \leq ||x|| + ||y||.$$

2. Show that $||x|| = ||-x||$.

3. For each natural number n, let $d_k(n)$ be the number of solutions of $x_1 \cdots x_k = n$ with each x_i a positive integer. Show by induction (or otherwise) that

$$d_k(n) = \sum_{d|n} d_{k-1}(d), \qquad k \geq 2.$$

4. Show that for any $\epsilon > 0$, $d_k(n) = O(n^\epsilon)$ where the implied constant depends only on k.

5. Applying Weyl's inequality, show that

$$S(q, a) = \sum_{r=1}^{q} e(ar^k/q) = O(q^{1-1/2^{k-1}+\epsilon}),$$

for any $\epsilon > 0$.