1. Compute the Fourier transform of \( g(x) = 1 - |x| \) for \( |x| \leq 1 \) and zero otherwise. Using this, apply the Poisson summation formula to deduce
\[
\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}.
\]

2. Suppose \( f \) is a continuous function of \( \mathbb{R}^1 \) with period 1. Prove that
\[
\lim_{N \to \infty} \frac{1}{N} \sum_{n=1}^{N} f(n\alpha) = \int_{0}^{1} f(t) \, dt
\]
for every irrational real number \( \alpha \). [Hint: do it first for \( f(t) = e^{2\pi i kt} \), with \( k = 0, \pm 1, \pm 2, \ldots \)]

3. Let \( w \in \mathbb{C} \) be a fixed complex number with \( |w| < 1 \). Let
\[
f(z) = \frac{z - w}{1 - wz}.
\]
Show that \( f \) is regular in \( |z| \leq 1 \) and calculate \( f(w) \) and \( f'(w) \).

4. Find the radius of convergence of
\[
\sum_{n=1}^{\infty} \frac{z^n}{n^n}.
\]

5. Prove the following discrete version of integration by parts: For any two sequences of numbers \( a_n, b_n \),
\[
S_N := \sum_{n=1}^{N} a_n b_n = a_N B_N - \sum_{n=1}^{N-1} B_n (a_{n+1} - a_n),
\]
where
\[
B_n = \sum_{j=1}^{n} b_j.
\]
Using this result, show that the power series
\[
\sum_{n=1}^{\infty} \frac{z^n}{n^n}
\]
converges for every complex number \( z \) with \( |z| = 1 \), and \( z \neq 1 \).

6. Show that the series
\[
\sum_{n=1}^{\infty} \frac{z^n}{n(n+1)}
\]
converges for all \( |z| \leq 1 \). What is its radius of convergence? Does the series
\[
\sum_{n=1}^{\infty} z^n
\]
converge for any \( z \) with \( |z| = 1 \)?

7. Let \( n \) be a non-negative integer. Define for \( \alpha \in \mathbb{C} \), the binomial coefficient
\[
\binom{\alpha}{n} = \frac{\alpha(\alpha - 1) \cdots (\alpha - n + 1)}{n!}.
\]
Show that the series
\[
\sum_{n=0}^{\infty} \binom{\alpha}{n} z^n
\]
converges for \( |z| < 1 \).