1. Determine the number of zeros of the polynomial
\[ 2z^5 - 6z^2 + z + 1 = 0 \]
in the annulus \( 1 \leq |z| \leq 2 \).

2. Let \( f \) be a meromorphic function and \( A \) its set of poles. Suppose it has a finite number of poles all of which lie in the upper half-plane (that is, have positive imaginary part). Suppose there is a constant \( K > 0 \) such that for some fixed \( \delta > 1 \), we have
\[ |f(z)| \leq K/|z|^\delta, \]
for \( |z| \) sufficiently large. Show that
\[ \int_{-\infty}^{\infty} f(x)dx = 2\pi i \sum_{a \in A} \text{Res}(f; a). \]

3. Show that
\[ \int_{-\infty}^{\infty} \cos x \frac{dx}{x^2 + 1} = \frac{\pi}{e}. \]

4. Let \( f(z) \) have a simple pole at \( z = 0 \). Let \( C(\epsilon) \) be the semicircular arc from \( -\epsilon \) to \( \epsilon \) of radius \( \epsilon > 0 \). Show that
\[ \lim_{\epsilon \to 0} \int_{C(\epsilon)} f(z)dz = -\pi i \text{Res}(f; 0). \]
[Note: \( C(\epsilon) \) is not a closed path.]

5. Let \( f \) be meromorphic on \( \mathbb{C} \) having only a finite number of poles all of which are simple. Suppose that there is a constant \( K > 0 \) such that
\[ |f(z)| \leq K/|z| \]
for \( |z| \) sufficiently large. Let \( t > 0 \). Prove that
\[ \int_{-\infty}^{\infty} f(x)e^{itx}dx = 2\pi i \sum_{a \in A} \text{Res}(f(z)e^{itz}; a) + \pi i \sum_{b \in B} \text{Res}(f(z)e^{itz}; b), \]
where \( B \) is the set of poles of \( f \) lying on the real axis and \( A \) is the set of poles lying in the upper half-plane.
6. Compute
\[ \int_{-\infty}^{\infty} \left( \frac{\sin x}{x} \right)^2 e^{itx} \, dx \]
for real \( t > 0 \).

7. Suppose \( \alpha \) is a complex number, \( |\alpha| \neq 1 \). Compute
\[ \int_{0}^{2\pi} \frac{d\theta}{1 - 2\alpha \cos \theta + \alpha^2}. \]

8. Let \( x > 0, x \neq 1 \). Show that
\[ \frac{1}{2\pi i} \int_{2-i\infty}^{2+i\infty} \frac{x^s}{s} \, ds = \begin{cases} 1 & \text{if } x > 1 \\ 0 & \text{if } 0 < x < 1 \end{cases} \]
What is the value of the integral if \( x = 1 \)?

9. Let \( \Gamma(s) \) denote the \( \Gamma \)-function. Calculate
\[ \text{Res}(\Gamma(s); -n) \]
for \( n = 0, 1, 2, ... \).

10. Show that
\[ \prod_{n=2}^{\infty} \left( 1 - \frac{1}{n^2} \right) = \frac{1}{2}. \]