Math 227 Queen's University, Department of Mathematics

Vector Analysis, Homeworkz 6

October 2013

1. Calculate the area of the bounded region inside the folium of Descartes, $x^3 + y^3 = 3xy$.

a)Sketch the bounded region and show that this region has a boundary which is parameterized by the vector function $\vec{r}(t) : [0, \infty) \to \mathbb{R}^2$

$$\vec{r}(t) = \frac{3t}{1+t^3}\vec{\mathbf{i}} + \frac{3t^2}{1+t^3}\vec{\mathbf{j}}$$

b) Using this parameterization and Green's Theorem calculate the area of the bounded region.

2. Let $\vec{F} = (3x^2y + y^3 + e^x)\,\vec{\mathbf{i}} + \left(e^{y^2} + 12x\right)\vec{\mathbf{j}}$. Consider the line integral of $\vec{F}$ around the circle of radius a, centered at the origin and oriented counterclockwise.

a) Find the line integral for a=1.

b) For which value of a is the line integral a maximum. Give a clear explanation of your conclusion.

3. The electric field $\vec{E}$, at the point with position vector $\vec{r}$ in $\mathbb{R}^3$, due to a charge q at the origin is given by

$$\vec{E}(\vec{r}) = q\frac{\vec{r}}{\|\vec{r}\|^3},$$

a) Compute curl $\vec{E}$. Is $\vec{E}$ a path independent vector field? Give a clear explanation

of your conclusion.

b) If possible, find a potential function for $\vec{E}$.