

## Bond valuation

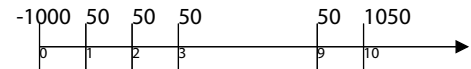
### MATH 384, Review lecture 6

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## Example

- Suppose that a bond is purchased for the face value of \$1,000 and is redeemed according to the following time diagram:



- What is the investor's yield rate?
- Answer: 5%
- Periodic (bond interest) payments are called *coupons*.

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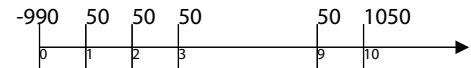
## In this lecture

- Bond price quotations in the press
- Terminology and examples
- Price on a coupon date
- Bond as an amortized loan: schedule

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## Example

- Suppose now that the bond is purchased for \$990 instead of \$1000 (bonds can be traded, and the price is determined by a number of factors such as risk, market situation).



- If the actual price is less than the face value, then the bond is purchased **at a discount**, \$10 in this example.
- What is the yield on the investment?
- Answer: 5.13%

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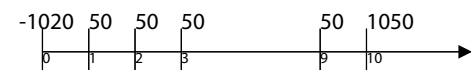
## Bond

- A **bond** is an interest-bearing security which promises to pay a stated amount (or amounts) of money at some future date (or dates).
- Bonds are used by corporations, governments, municipalities to raise funds.
- Bonds are redeemed at the end of a fixed period of time, called **term** of the bond. The end of the term is called **maturity date**.

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## Example

- Suppose, on the other hand, that the bond is purchased for \$1,020 instead of \$1000.



- If the actual price is greater than the face value, then the bond is purchased **at a premium**, \$20 in this example.
- What is the investor's yield in this case?
- Answer: 4.74%.

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## Definitions

- Face Amount is the face value of the bond
- Coupon Rate is the bond interest rate
- Maturity Date is the date where the final value is paid
- Term to Maturity*, sometimes specified in stead of maturity date, is period over which the bonds *coupons* (bond interest payments) are to be paid
- Redemption Amount is the final value paid on the maturity date
- In most cases (and in this class) the face amount and the redemption amount will be the same

## Example

- As the price goes up, the yield goes down, and vice versa.

## Assumptions

- Different types of bonds are used; this lecture only considers bonds with a fixed maturity date and fixed payments.
- We do not consider the probabilities of default.

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## Problem types

We have two computational problems appearing in practice:

- Find the price, given the yield rate
- Find the yield rate, given the price

We will consider not only the price  $P$  (the price at issue), but also the price *immediately after* each coupon payment.

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## Formal notations

- $P$  = the price of a bond
- $F$  = the Face Value, or face amount
- $r$  = the coupon rate, rate per coupon payment period (six months unless specified)
- $n$  = the number of coupon payment periods
- $K = Fv^n$ , the present value of the face value repayment.

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## Basic formula

$$\begin{aligned} P &= Fra_{\overline{n}|r} + Fv^n \\ &= Fra_{\overline{n}|r} + K \end{aligned} \quad (1)$$

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## Formal notations

- $i$  = the yield rate of the bond, that is, the rate realized by the investor, or the internal rate of return, or the yield to maturity.
- $j$  is used in place of  $i$  when dealing with discussing coupon periods that are not annual
- For example, if the coupons are paid semiannually and the bond yields 10% annual nominal rate convertible semiannually, then  $j = 5\% = 0.05$ .

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## Example

**Problem 1** An investor buys a 5-year U.S. Treasury note to realize the annual nominal rate of interest, convertible semiannually, of 3.490%. The par value of the note is \$100, the nominal yield is 3.375%. The coupons are paid semiannually.

Find the price of the note. Show 8 digits.

Note: the issue date of this note is 10-15-2004, the maturity date is 10-15-2009. The information is obtained from the

following web site: <http://www.publicdebt.treas.gov/AI/OFNtebnd>

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## Formal notations

- Values of  $F$ ,  $C$ ,  $r$ ,  $n$  are fixed throughout the term of the bond. They determine the timing and the amount of payments.
- Values of  $P$ ,  $i$  (or  $j$ ), and  $K$  may vary with time.

## Example

The coupon rate is  $r = 3.375\%/2$ . The yield rate is  $j = 3.490\%/2$ . There are  $n = 10$  coupon payments, redemption value is  $F = 100$ . Using the formula above, we have:

$$P = 100 \cdot \frac{0.03375}{2} a_{\overline{10}|0.0349/2} + \left(1 + \frac{0.0349}{2}\right)^{-10} \cdot 100 = 99.476542. \quad (2)$$

Answer: \$99.476542

## Premium/discount formula

$$\begin{aligned}
 P &= F + (Fr - Fj)a_{\overline{m}|} \\
 &= Fra_{\overline{m}|} + F(r - j)a_{\overline{m}|}
 \end{aligned}
 \tag{3}$$

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## Bond amortization schedule

Assume:

- The redemption value  $F = 1$
- Therefore the coupon payment is  $Fr$ .
- The price  $P = 1 + p$ , where  $p$  is the premium/discount.
- Book value is taken immediately after the coupon payment.
- The rate of interest  $i$  is not necessarily annual; it is the effective rate of interest for the coupon payment period.

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## Makeham formula

Letting  $a_{\overline{m}|} = \frac{1-v^n}{j}$

$$\begin{aligned}
 P &= Fv^n + Fra_{\overline{m}|} \\
 &= F + F(r - j)a_{\overline{m}|} \\
 &= K + \frac{r}{j}(F - K)
 \end{aligned}
 \tag{4}$$

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## Bond amortization schedule

	Cpn	Interest earned	Principal adjustmt	Book value
0				$1 + (r - i)a_{\overline{m} i}$
1	$r$	$i(1 + (r - i)a_{\overline{m} i})$	$(r - i)v^n$	$1 + (r - i)a_{\overline{m-1} i}$
2	$r$	$i(1 + (r - i)a_{\overline{m-2} i})$	$(r - i)v^{n-1}$	$1 + (r - i)a_{\overline{m-2} i}$
...	...	...	...	...
$t$	$r$	$i(1 + (r - i)a_{\overline{m-t+1} i})$	$(r - i)v^{n-t+1}$	$1 + (r - i)a_{\overline{m-t} i}$
...	...	...	...	...
$n$	$r$	$i(1 + (r - i)a_{\overline{1} i})$	$(r - i)v$	1
	$nr$	$nr - p$	$p = (r - i)a_{\overline{m} i}$	

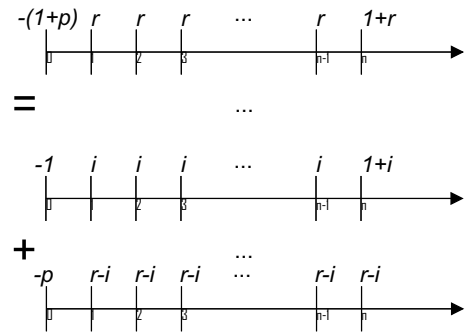
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## Example

**Problem 2** Suppose that a 5-year bond with par value \$1000 with annual coupon payments, coupon rate 20% is bought for \$756.44. Calculate the yield rate.  
Hint: the yield rate is a multiple of 10%.

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## Understanding the table



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## Example

Answer: 30%

## Understanding the table

- The net present value of each of the series of payments is 0 at the rate  $i$ .
- Note: principal adjustment is also called **premium amortization**.

## Examples

**Problem 3** Among a company's assets and accounting records, an actuary finds a 15-year bond that was purchased at a premium. From the records, the actuary has determined the following:

1. The bond pays semi-annual interest.
2. The amount for amortization of the premium in the 2nd coupon payment was 977.19
3. The amount for amortization of the premium in the 4th coupon payment was 1046.79

Find the premium.

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## Examples

**Problem 5** A 1000 par value 20-year bond with annual coupons and redeemable at maturity at 1050 is purchased for  $P$  to yield an annual effective rate of 8.25%. The first coupon is 75. Each subsequent coupon is 3% greater than the preceding coupon. Determine  $P$ .

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## Examples

Answer: 48,739

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## Examples

Answer: 1115

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## Examples

**Problem 4** A 10,000 par value 10-year bond with 8% annual coupons is bought at a premium to yield an annual effective rate of 6%. Calculate the interest portion of the 7th coupon.

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## Debt markets

- A report on total size of debt outstanding in the U.S. is here: <http://www.bondmarkets.com/story.asp?id=98>
- To compare, the GDP in year 2003 was \$11.0T

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## Examples

Answer: 642

## Summary

- Reading: Chapter 4 through 4.3.1.
- Sample test problems are on the web site.