

Bond valuation

MATH 384, Review lecture 6

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In this lecture

- Bond price quotations in the press
- Terminology and examples
- Price on a coupon date
- Bond as an amortized loan: schedule

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Bond

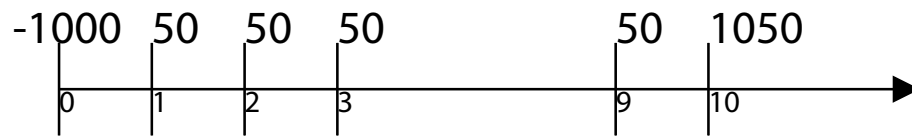
- A **bond** is an interest-bearing security which promises to pay a stated amount (or amounts) of money at some future date (or dates).
- Bonds are used by corporations, governments, municipalities to raise funds.
- Bonds are redeemed at the end of a fixed period of time, called **term** of the bond. The end of the term is called **maturity date**.

Definitions

- Face Amount is the face value of the bond
- Coupon Rate is the bond interest rate
- Maturity Date is the date where the final value is paid
- *Term to Maturity*, sometimes specified in stead of maturity date, is period over which the bonds *coupons* (bond interest payments) are to be paid
- Redemption Amount is the final value paid on the maturity date
- In most cases (and in this class) the face amount and the redemption amount will be the same

Example

- Suppose that a bond is purchased for the face value of \$1,000 and is redeemed according to the following time diagram:



- What is the investor's yield rate?
- Answer: 5%
- Periodic (bond interest) payments are called *coupons*.

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Example

- Suppose now that the bond is purchased for \$990 instead of \$1,000 (bonds can be traded, and the price is determined by a number of factors such as risk, market situation).

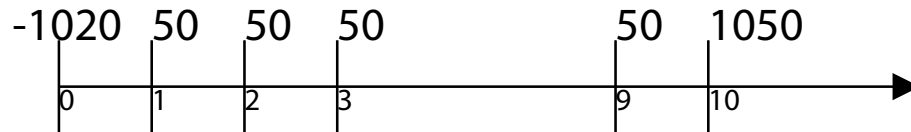


- If the actual price is less than the face value, then the bond is purchased **at a discount**, \$10 in this example.
- What is the yield on the investment?
- Answer: 5.13%

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Example

- Suppose, on the other hand, that the bond is purchased for \$1,020 instead of \$1000.



- If the actual price is greater than the face value, then the bond is purchased **at a premium**, \$20 in this example.
- What is the investor's yield in this case?
- Answer: 4.74%.

Example

- As the price goes up, the yield goes down, and vice versa.

Assumptions

- Different types of bonds are used; this lecture only considers bonds with a fixed maturity date and fixed payments.
- We do not consider the probabilities of default.

Formal notations

- P = the **price** of a bond
- F = the **Face Value**, or **face amount**
- r = the **coupon rate**, rate per coupon payment period (six months unless specified)
- n = the number of coupon payment periods
- $K = Fv^n$, the present value of the face value repayment.

Formal notations

- i = the **yield rate** of the bond, that is, the rate realized by the investor, or the **internal rate of return**, or the **yield to maturity**.
- j is used in place of i when dealing with discussing coupon periods that are not annual
- For example, if the coupons are paid semiannually and the bond yields 10% annual nominal rate convertible semiannually, then $j = 5\% = 0.05$.

Formal notations

- Values of F , C , r , n are fixed throughout the term of the bond. They determine the timing and the amount of payments.
- Values of P , i (or j), and K may vary with time.

Problem types

We have two computational problems appearing in practice:

- Find the price, given the yield rate
- Find the yield rate, given the price

We will consider not only the price P (the price at issue), but also the price *immediately after* each coupon payment.

Basic formula

$$\begin{aligned} P &= F r a_{\overline{n}|} + F v^n \\ &= F r a_{\overline{n}|} + K \end{aligned} \tag{1}$$

Example

Problem 1 An investor buys a 5-year U.S. Treasury note to realize the annual nominal rate of interest, convertible semiannually, of 3.490%. The par value of the note is \$100, the nominal yield is 3.375%. The coupons are paid semiannually.

Find the price of the note. Show 8 digits.

Note: the issue date of this note is 10-15-2004, the maturity date is 10-15-2009. The information is obtained from the following web site: <http://www.publicdebt.treas.gov/AI/OFNtebnd>

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Example

The coupon rate is $r = 3.375\%/2$. The yield rate is $j = 3.490\%/2$. There are $n = 10$ coupon payments, redemption value is $F = 100$. Using the formula above, we have:

$$P = 100 \cdot \frac{0.03375}{2} a_{\overline{10}|0.0349/2} + \left(1 + \frac{0.0349}{2}\right)^{-10} \cdot 100 = 99.476542. \quad (2)$$

Answer: \$99.476542

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Premium/discount formula

$$\begin{aligned} P &= F + (Fr - Fj)a_{\overline{n}|} \\ &= Fra_{\overline{n}|} + F(r - j)a_{\overline{n}|} \end{aligned} \quad (3)$$

Makeham formula

Letting $a_{\overline{n}|} = \frac{1-v^n}{j}$

$$\begin{aligned} P &= Fv^n + Fra_{\overline{n}|} \\ &= F + F(r - j)a_{\overline{n}|} \\ &= K + \frac{r}{j}(F - k) \end{aligned} \quad (4)$$

Example

Problem 2 *Suppose that a 5-year bond with par value \$1000 with annual coupon payments, coupon rate 20% is bought for \$756.44. Calculate the yield rate. Hint: the yield rate is a multiple of 10%.*

Example

Answer: 30%

Bond amortization schedule

Assume:

- The redemption value $F = 1$
- Therefore the coupon payment is Fr .
- The price $P = 1 + p$, where p is the premium/discount.
- Book value is taken immediately after the coupon payment.
- The rate of interest i is not necessarily annual; it is the effective rate of interest for the coupon payment period.

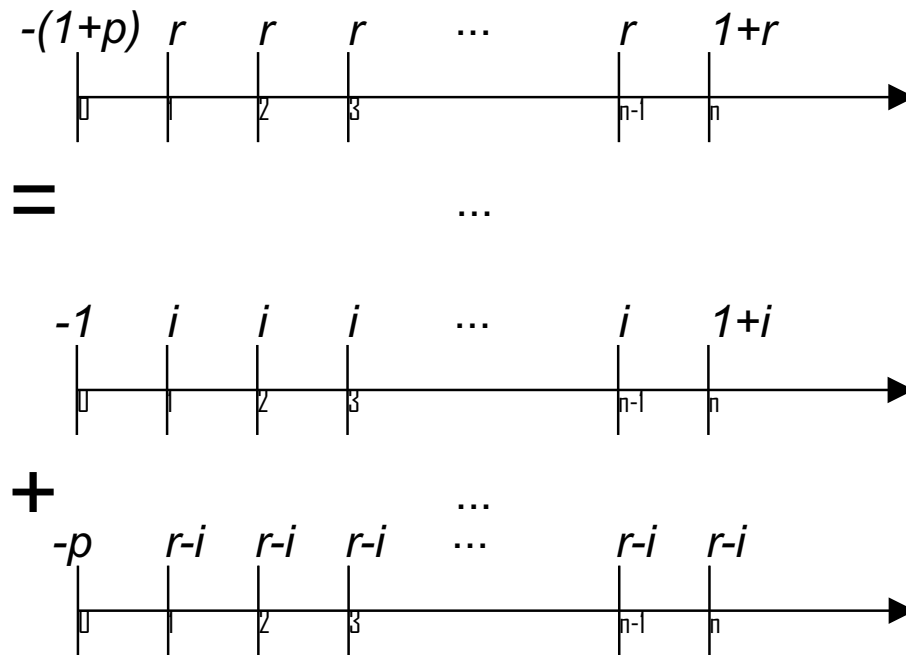
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Bond amortization schedule

| | Cpn | Interest earned | Principal adjustmt | Book value |
|-----|------|--|---------------------------------|-----------------------------------|
| 0 | | | | $1 + (r - i)a_{\overline{n} i}$ |
| 1 | r | $i(1 + (r - i)a_{\overline{n-1} i})$ | $(r - i)v^n$ | $1 + (r - i)a_{\overline{n-1} i}$ |
| 2 | r | $i(1 + (r - i)a_{\overline{n-2} i})$ | $(r - i)v^{n-1}$ | $1 + (r - i)a_{\overline{n-2} i}$ |
| ... | ... | ... | ... | ... |
| t | r | $i(1 + (r - i)a_{\overline{n-t+1} i})$ | $(r - i)v^{n-t+1}$ | $1 + (r - i)a_{\overline{n-t} i}$ |
| ... | ... | ... | ... | ... |
| n | r | $i(1 + (r - i)a_{\overline{1} i})$ | $(r - i)v$ | 1 |
| | nr | $nr - p$ | $p = (r - i)a_{\overline{n} i}$ | |

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Understanding the table



Understanding the table

- The net present value of each of the series of payments is 0 at the rate i .
- Note: principal adjustment is also called **premium amortization**.

Examples

Problem 3 *Among a company's assets and accounting records, an actuary finds a 15-year bond that was purchased at a premium. From the records, the actuary has determined the following:*

- 1. The bond pays semi-annual interest.*
- 2. The amount for amortization of the premium in the 2nd coupon payment was 977.19*
- 3. The amount for amortization of the premium in the 4th coupon payment was 1046.79*

Find the premium.

Examples

Answer: 48,739

Examples

Problem 4 *A 10,000 par value 10-year bond with 8% annual coupons is bought at a premium to yield an annual effective rate of 6%. Calculate the interest portion of the 7th coupon.*

Examples

Answer: 642

Examples

Problem 5 *A 1000 par value 20-year bond with annual coupons and redeemable at maturity at 1050 is purchased for P to yield an annual effective rate of 8.25%. The first coupon is 75. Each subsequent coupon is 3% greater than the preceding coupon.
Determine P .*

Examples

Answer: 1115

Debt markets

- A report on total size of debt outstanding in the U.S. is here: <http://www.bondmarkets.com/story.asp?id=98>
- To compare, the GDP in year 2003 was \$11.0T

Summary

- Reading: Chapter 4 through 4.3.1.
- Sample test problems are on the web site.