MATH 328
REAL ANALYSIS
WINTER 2010

Assignment 1 (due January 21)

(1) What is the negation of the following statement:

$$\forall \epsilon > 0 \quad \exists N \in \mathbb{N} \quad \forall n \in \mathbb{N} : n \geq N \Rightarrow |\frac{1}{n} - 1| \leq \epsilon$$

Is the statement true or false.

(2) (a) Give a proof by contraposition of the following statement: Let n be a natural number. If n^2 is even, then n is even.

(b) Prove by cases the following statement: There are two irrational numbers a and b such that a^b is rational. (Hint: Look at $\sqrt{2}^{\sqrt{2}}$.)

(3) (a) Prove that $\sqrt[3]{5}$ is not rational.

(b) Show that $\sqrt[3]{5}$ exists as a real number, by using the least upper bound principle.

(4) Let

$$\mathcal{P}(\mathbb{N}) := \{A \mid A \subset \mathbb{N}\}$$

be the power set of the natural numbers. Show that $\mathcal{P}(\mathbb{N})$ is uncountable. (Prove this directly by a diagonal argument, not by using the fact that the real numbers are uncountable.)