Chapter, section and problem numbers refer to the 3rd edition of the Ghahramani textbook.

Four of the following six problems will be chosen at random to be marked.

1. (a) Can an event $A$ be independent of itself, according to the mathematical definition of independence? If so, determine when this can happen. If not, prove that it cannot happen.

   (b) If the events $A$ and $B$ are independent, and the events $A$ and $C$ are independent, is it true that the events $A$ and $B - C$ are also independent? Prove, or give a counterexample.

2. Section 3.5, # 11

3. Section 3.5, # 22

4. Section 3.5, # 29

5. A monkey at a typewriter types each of the 26 capital letters of the alphabet exactly once, the order being random.

   (a) What is the probability that the word “HAMLET” appears somewhere in the string of letters?

   (b) How many independent monkey typists would you need in order that the probability that the word “HAMLET” appears at least once is greater than 0.9?

6. For the sake of argument, suppose that an aircraft engine fails in flight with probability 0.4, independently of the plane’s other engines. Also, suppose that a plane can complete its journey successfully if at least half of its engines do not fail. Under such circumstances, would you prefer to fly on a four-engine plane or a two-engine one?

   **Bonus question:** Let $S = \{1, 2, \ldots, n\}$ and suppose that $A$ and $B$ are, independently, equally likely to be of any of the $2^n$ subsets (including $S$ and the empty set) of $S$.

   (a) Find $P(A \subset B)$.

   *Hint:* Condition on the events $N(B) = i$ that $B$ has $i = 0, 1, 2, \ldots, n$ elements.

   (b) Find $P(AB = \emptyset)$.

   (Bonus questions do not have to be attempted, but bonus marks will be awarded for a correct solution.)