Chapter, section and problem numbers refer to the 3rd edition of the Ghahramani textbook.

Four of the following six problems will be chosen at random to be marked.

1. Section 4.2, # 2. Also, determine the distribution function of \(X\).

2. Section 4.2, # 12.

3. Let \(X\) be a real number selected at random from the interval \([0, 1]\).
   
   (a) Calculate \(P\left(\frac{X}{1+X} \leq 2/5\right)\).
   
   (b) Let \(Y = X/(1+X)\). Determine the distribution function, \(F_Y\), of \(Y\), and sketch its graph.

4. Suppose an urn contains three balls, one black, one white, and one green. Assume that balls are repeatedly drawn, one at a time, with replacement and let \(X\) denote the number of draws until each color appears at least once.
   
   (a) Find the probabilities \(P(X > n)\) for \(n = 0, 1, 2, \ldots\)
   
   (b) Find the pmf of \(X\).

5. A gambling book recommends the following strategy for the game of roulette. The gambler should bet $1 on red. If red appears (which has probability \(18/38\)), then the gambler should take her $1 profit and quit. If the gambler loses this bet (which has probability \(20/38\)), she should make additional $1 bets on red on each of the next two spins of the wheel and then quit. Let \(X\) denote the gambler’s net winnings when she quits.
   
   (a) Find the probability mass function of \(X\) and \(P(X > 0)\).
   
   (b) Find \(E[X]\).

6. Section 4.4, # 12.

**Bonus question:**

(a) Let \(X\) be a discrete random variable which takes values in the set of positive integers \(\{1, 2, 3, \ldots\}\). Show that

\[
E[X] = \sum_{k=0}^{\infty} P(X > k) \tag{1}
\]
In the rest of this problem, we assume the following setup. A box initially contains 1 red and 1 black ball. A ball is drawn at random from the box, and two balls of the same color are put back in the box. This procedure is performed repeatedly: start with 1 red, 1 black; draw a ball, replace with two of the same color; draw another ball, replace with two of the same color; draw another ball, and so on.

Let $X$ be the number of draws until a black ball is drawn for the first time. Thus, for example, $X = 3$ implies that the first two draws were reds and the third draw was a black.

(b) Find $P(X > k)$. [Hint: what must happen on the first $k$ draws for $X > k$ to be true?]

(c) Use part (a) to determine $E[X]$.

(Bonus questions do not have to be attempted, but bonus marks will be awarded for a correct solution.)