Chapter, section and problem numbers refer to the 3rd edition of the Ghahramani textbook.

Four of the following six problems will be chosen at random to be marked.

1. Section 7.1, # 7.

2. Suppose the lifetime of a light bulb produced in a certain factory is an exponentially distributed random variable. We know that 50% of these light bulbs die within one year.
   (a) Find the probability that a newly purchased light bulb will last more than 2 years?
   (b) If a light bulb has been working for a year, what is the probability that it will not survive another year.

3. Section 7.3 # 5.

4. Let \( X \) be a normally distributed random variable with mean 0 and variance 4.
   (a) Let \( Y = \lfloor |X| \rfloor \), where \( \lfloor a \rfloor \) is the integer part of \( a \) (i.e., the greatest integer less than or equal to \( a \)). Find \( P(Y = 0) \) and \( P(Y = 1) \).
   (b) What is the probability that the interval \((X - 1, X + 1)\) contains the value 0?

5. Section 7.2, # 16.

6. Section 7.2, # 27. (Use the normal approximation to the binomial distribution.)

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**Bonus question:**

Let \( \{N(t), \ t \geq 0\} \) denote a Poisson process with rate \( \lambda > 0 \). For \( n \geq 1 \), define \( T_n \) to be the time of the occurrence of the \( n \)th event.

(a) Determine the distribution function, \( F(t) \), of the random variable \( T_n \).
   \( \textbf{[Hint: Express the event } "T_n > t" \text{ in terms of } N(t).\]}

(b) Show that the density function of \( T_n \) is given by

\[
f(t) = \begin{cases} 
\lambda e^{-\lambda t} \frac{(\lambda t)^{n-1}}{(n-1)!} & \text{if } t \geq 0 \\
0 & \text{otherwise.}
\end{cases}
\]

(Bonus questions do not have to be attempted, but bonus marks will be awarded for a correct solution.)