Chapter, section and problem numbers refer to the 3rd edition of the Ghahramani textbook.

Four of the following six problems will be chosen at random to be marked.

1. Section 7.1, # 9.
2. Section 7.3, # 4.
3. Section 7.2, # 12.
5. Let $X$ be a normally distributed random variable with mean 0 and variance 4.
   
   (a) Determine $P (X(X-1) > 2)$.
   (b) Determine the $a$ for which $P(X^4 > a) = 0.25$.

6. Xavier and Yasmin each toss a coin 100 times. The coin is biased so that the probability that it comes up heads on a toss is 2/3. The tosses can be assumed to be independent of each other.
   
   (a) Find an exact expression for the probability that Xavier and Yasmin each get between 50 and 75 heads (both inclusive).
   (b) Use the normal approximation of the deMoivre-Laplace theorem to evaluate the expression obtained in part (a).

---

**Bonus question:** Recall that a random variable $X$ is *memoryless* if for any $s, t \geq 0$,

$$P(X > s + t|X > t) = P(X > s).$$

(It is tacitly assumed in this definition that $P(X > t) > 0$ for all $t$ so that the conditional probability is well defined.)

(a) Show that a memoryless random variable is always “positive” in the sense that $P(X \leq 0) = 0$. 


(b) Show that if \( X \) is a memoryless random variable, then its distribution function is in the form 
\[
F(t) = 1 - e^{-\lambda t}, \quad t \geq 0,
\]
where \( \lambda \) is a positive constant. Thus exponential random variables are the only random variables with the memoryless property (*).

**Hint:** Let \( G(t) = 1 - F(t) \). Show that \( G(m/n) = [G(1)]^{m/n} \) for all positive integers \( n \) and \( m \), and that \( 0 < G(1) < 1 \). Then approximate any real \( t \geq 0 \) from above by a sequence of rational numbers \( t_n \).

(Bonus questions do not have to be attempted, but bonus marks will be awarded for a correct solution.)