Section and problem numbers refer to the 3rd edition of the Ghahramani textbook.

1. Show that the Gamma\((r, \lambda)\) probability density function is maximized at \((r - 1)/\lambda\) for \(r > 1\). What happens if \(0 < r \leq 1\)?

2. Let \(X\) and \(Y\) be independent random variables, each with a normal distribution with mean 0 and standard deviation \(\sigma\). If the point \((X, Y)\) is placed on the plane, what is the probability that the distance of this point from the origin is more than 1.5\(\sigma\)?

3. Let \(X\) have a Gamma\((m/2, 1/2)\) distribution and let \(Y\) have a Gamma\((n/2, 1/2)\) distribution, where \(m\) and \(n\) are positive integers, and suppose that \(X\) and \(Y\) are independent. Let \(U = \frac{X}{m}/\frac{Y}{n}\). Show that the probability density function of \(U\) is given by

\[
  f(u) = \begin{cases} 
    \frac{m}{nB(m/2,n/2)} \cdot \frac{(mu/n)^{(m-2)/2}}{[1 + (mu/n)]^{(m+n)/2}}, & \text{if } u > 0, \\
    0, & \text{if } u \leq 0.
  \end{cases}
\]

This is called the \(F\)-distribution with \(m\) and \(n\) degrees of freedom. \textit{Hint:} Define \(V = Y\) and find the joint density function of \(U\) and \(V\) using the methods of Section 8.4. Then from this find the marginal density function of \(U\).

4. Section 7.5, #8.

5. Let \(X\) have a Gamma\((r, \lambda)\) distribution and let \(Y\) have a Beta\((\alpha, \beta)\) distribution, where \(r = \alpha + \beta\). Suppose that \(X\) and \(Y\) are independent. Show that \(XY\) has a Gamma\((\alpha, \lambda)\) distribution.