Section and problem numbers refer to the 3rd edition of the Ghahramani textbook.

Four of the following five problems will be chosen at random to be marked.

1. Section 11.1, #9.

2. Section 11.1, #16.

3. Section 11.1, #24. *Hint:* Write out the power series expansion of $M_X(t)$.

4. Let $X_1, X_2, \ldots$ be independent and identically distributed standard normal random variables and let $N = N_0 + 1$, where $N_0$ has a Poisson distribution with parameter $\lambda$. Also suppose that $N_0$ is independent of the $X_i$’s. By conditioning on $N$, show that the moment generating function of the random sum $Y = X_1 + \cdots + X_N$ is given by

$$M_Y(t) = \exp \left\{ \lambda \left( e^{t^2/2} - 1 \right) + \frac{t^2}{2} \right\}.$$

5. Let $X = (X_1, X_2)^T$ have joint moment generating function

$$M_X(t) = e^{t_1 - t_2 + t_1^2 + t_2^2/2 - t_1 t_2/2},$$

where $t = (t_1, t_2)^T$. Find $E(X)$ and $Cov(X)$. 