Queen’s University  
Department of Mathematics and Statistics  

**MTHE/STAT 353**  
Midterm Examination   February 26, 2016

- Total points = 30. Duration = 58 minutes.
- This is a closed book exam.
- One 8.5 by 11 inch sheet of notes, written on both sides, is permitted.
- A simple calculator is permitted.
- Write the answers in the space provided, continue on the backs of pages if needed.
- **SHOW YOUR WORK CLEARLY.** Correct answers without clear work showing how you got there will not receive full marks.
- Marks per part question are shown in brackets at the right margin.
- The last page contains formulas you may find useful. Please check this page first.

**Marks:** Please do not write in the space below.

Problem 1 [10]

Problem 2 [10]

Problem 3 [10]

Total: [30] __________
1. Let $X$ and $Y$ be jointly discrete random variables with joint pmf

$$p(x, y) = K \frac{(\lambda(1 - p))^y \left(\frac{p}{1-p}\right)^x}{x!(y-x)!},$$

for $y = 0, 1, 2, \ldots$ and $x = 0, 1, \ldots, y$, and $p(x, y) = 0$ otherwise. Here $\lambda > 0$ and $p \in (0, 1)$ are both fixed parameters, and $K$ is a normalizing constant.

(a) Find $K$. [5]
(b) Find the marginal pmf of $X$. [5]
2. A system has 4 components. When power is applied to the system it takes some time for each component to initialize. Let $X_i$ be the time, in seconds, for component $i$ to initialize, for $i = 1, 2, 3, 4$. Suppose the $X_i$ are independent, continuous random variables, each with pdf

$$f_X(x) = \begin{cases} 
3x^2/b^3 & \text{for } 0 \leq x \leq b \\
0 & \text{otherwise},
\end{cases}$$

where $b > 0$ is fixed. The system fails to initialize if and only if 2 or more components take longer than $b/2$ seconds to initialize. Find the probability that the system fails to initialize.
3. Let $X_1, \ldots, X_{10}$ be independent Uniform(0,1) random variables.

(a) Find the probability that all the $X_i$'s are greater than 0.5. [5]

(b) Find the probability that 2 of the $X_i$'s are in $[0, .2)$, 3 of the $X_i$'s are in $(.2, .7)$, and 5 of the $X_i$'s are in $[.7, 1]$. [5]
Formulas:

- The Uniform(0,1) distribution has pdf

\[ f(x) = \begin{cases} 
1 & \text{for } 0 < x < 1 \\
0 & \text{otherwise.}
\end{cases} \]